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CollegeBoard AP 50 YEARS

Setting a Policy for AP® Calculus

The purpose of this guide is to provide college faculty and administrators with research data, participation and performance data of AP® Calculus students, curricular content, and sample exam questions to facilitate the establishment of appropriate credit and placement policies for AP Calculus.

The Advanced Placement Program® (AP) provides motivated students with the opportunity to take college-level courses while still in high school. Students demonstrate their mastery of the curriculum by taking AP Exams—35 exams, including two in Calculus, are available in 20 subject areas. In 2005, more than 1.2 million students took AP Exams worldwide. Of the 2.1 million AP Exams taken in 2005, about 241,000 exams were in Calculus (186,000 for Calculus AB and 55,000 for Calculus BC). More than 3,000 colleges and universities, including many international institutions, accept qualifying AP Exam scores for credit, placement, or both.

Throughout its 50-year history, the AP Program has maintained high standards of rigor in its courses and exams. Since its inception, AP has been a respected force in American education due to the critical involvement of college and university faculty members.

Mathematics Faculty Involvement in AP

College and university faculty members play a vital role in every stage of development of an AP course and exam, helping to ensure their high quality. Each AP discipline has its own Development Committee—composed of college and university professors and experienced AP teachers—that is responsible for creating the course guidelines and exam questions. College and university faculty members also serve as the Chief Readers, responsible for establishing the exam-scoring guidelines and overseeing the annual AP Reading of the free-response section for their academic discipline.

> "The ideal of AP Calculus is to provide a smooth transition from high school to college mathematics. The successful student should fit naturally into the calculus sequence in most colleges and universities. The content of AP Calculus matches very well with the content of the first calculus courses in many colleges. The AB course matches with the first semester course, and the BC course matches with the first and second semester courses. Students who complete AP Calculus and achieve grades of 3, 4, or 5 on an AP Calculus Examination are usually prepared to succeed in the second or third semester of college calculus."
>
> —Bernard L. Madison, AP Calculus former Chief Reader
> University of Arkansas

The College Board publication *AP and Higher Education* discusses the following topics at greater length: how to set an AP policy, AP research studies, the development of AP courses and exams, and the AP Exam scoring. For more information or to request a copy of this publication, please go to **apcentral.collegeboard.com/highered.**

How to Set an AP Policy

The College Board encourages higher education institutions to base their AP policy decisions on data and research, and recognizes that different institutions and departments will set different policies, based upon factors unique to their institution, student body, and academic discipline. The best way for colleges and universities to determine their AP credit and placement policies is to conduct their own research on the performance of AP and non-AP students at their own institution and in their own department.

Research on AP Calculus Student Performance

Research studies show that students who do well on an AP Exam are academically prepared to place out of a corresponding college course and move on to the next higher-level course in the discipline. See Tables 1 and 2 for data from a research study comparing AP and non-AP student performance in second and third college calculus courses or calculus-required mathematics courses.

Table 1: Student Performance in Second and Third College Calculus Courses or Calculus-Required Mathematics Courses
AP Calculus AB Students Versus Non-AP Students

	AP EXAM GRADE	SECOND COURSES		THIRD COURSES	
		GPA	PERCENT OF STUDENTS SCORING AN A OR B	GPA	PERCENT OF STUDENTS SCORING AN A OR B
AP Students Who Place Out of Intro. Course	AP 5	3.23	80	3.04	77
	AP 4	2.79	66	2.75	63
	AP 3	2.67	60	2.70	61
Students Who Complete Intro. Course	Non-AP	2.52	54	2.62	58

Table 2: Student Performance in Second and Third College Calculus Courses or Calculus-Required Mathematics Courses
AP Calculus BC Students Versus Non-AP Students

	AP EXAM GRADE	SECOND COURSES		THIRD COURSES	
		GPA	PERCENT OF STUDENTS SCORING AN A OR B	GPA	PERCENT OF STUDENTS SCORING AN A OR B
AP Students Who Place Out of Intro. Course	AP 5	3.66	93	3.38	88
	AP 4	3.24	84	2.88	72
	AP 3	2.88	69	2.93	73
	AP 2	2.53	56	2.66	63
Students Who Complete Intro. Course	Non-AP	2.51	54	2.74	64

Taking the AP course and exam stimulates further interest in the subject area and encourages deeper disciplinary knowledge.

Research studies show that students who take the AP Calculus Exams are significantly more likely to take further course work in mathematics than students who do not take the AP Exam. See Tables 3 and 4 for data from this research study.

Table 3: Additional College Mathematics Course Work
AP Calculus AB Students Versus Non-AP Students

	AP EXAM GRADE	PERCENT TAKING ADDITIONAL MATH COURSES	AVERAGE NUMBER OF COLLEGE MATH COURSES TAKEN
AP Calculus AB Students	AP 5	89	2.7
	AP 4	81	2.3
	AP 3	82	2.2
	AP 2	89	2.5
	AP 1	84	2.1
Non-AP Students	Non-AP	78	1.9

Table 4: Additional College Mathematics Course Work
AP Calculus BC Students Versus Non-AP Students

	AP EXAM GRADE	PERCENT TAKING ADDITIONAL MATH COURSES	AVERAGE NUMBER OF COLLEGE MATH COURSES TAKEN
AP Calculus BC Students	AP 5	88	2.8
	AP 4	79	2.2
	AP 3	81	2.2
	AP 2	85	2.3
	AP 1	89	2.4
Non-AP Students	Non-AP	77	1.8

PDF copies of these and other research studies can be found at apcentral.collegeboard.com/colleges/research.

In addition to research studies on AP student performance, the College Board conducts college comparability studies to measure the degree to which the AP courses and exams are equivalent in content and difficulty to corresponding college courses. The AP Exam scoring rubric is established so that the lowest composite score that earns an AP grade of 5 is equivalent to the average score earned by college students who received grades of A in a comparable course. The lowest score that earns an AP grade of 4 is equivalent to the average B, and the lowest score that earns an AP grade of 3 is equivalent to the average C.

The research that the College Board conducts is intended to help institutions and academic departments as they establish appropriate AP policies. AP Central® (apcentral.collegeboard.com), the College Board's online home for AP professionals, contains other resources that may assist in this process, including the Course Description, released exam questions, and sample student responses at different levels of ability.

For more information go to:
apcentral.collegeboard.com/calculusab/exam
apcentral.collegeboard.com/calculusbc/exam

AP Calculus Students, Courses, and Exams

Participation and Performance Data for AP Calculus Students in 2005

Total Number of Schools Offering AP Calculus AB: 11,183
Total Number of Schools Offering AP Calculus BC: 4,191

Table 5: AP Calculus AB Exam Score Distribution, 2005

EXAM GRADE	NUMBER OF EXAMINEES	% AT
Score of 5	38,539	20.7%
Score of 4	36,347	19.5%
Score of 3	33,006	17.7%
Score of 2	31,141	16.7%
Score of 1	46,959	25.2%
	185,992	100.0%

Table 6: AP Calculus BC Exam Score Distribution, 2005

EXAM GRADE	NUMBER OF EXAMINEES	% AT
Score of 5	23,877	43.9%
Score of 4	9,237	17.0%
Score of 3	10,929	20.1%
Score of 2	3,695	6.8%
Score of 1	6,677	12.3%
	54,415	100.0%

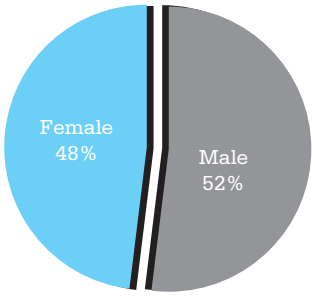


Figure 1: AP Calculus AB Examinees by Gender, 2005

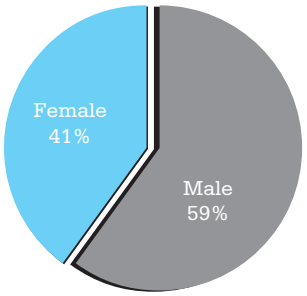


Figure 2: AP Calculus BC Examinees by Gender, 2005

Figure 3: AP Calculus AB Examinees by Race and Ethnicity, 2005

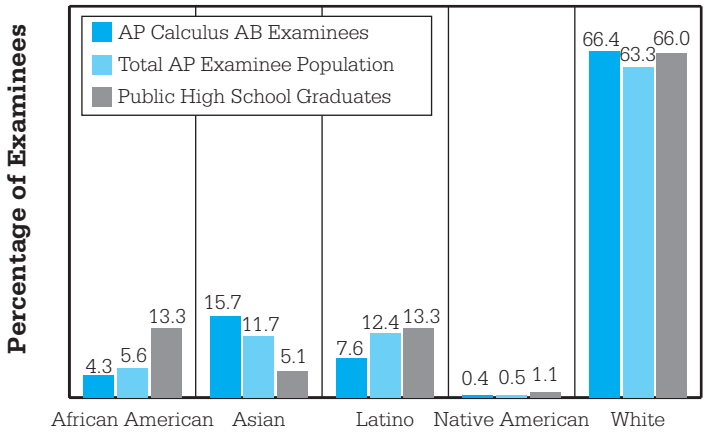
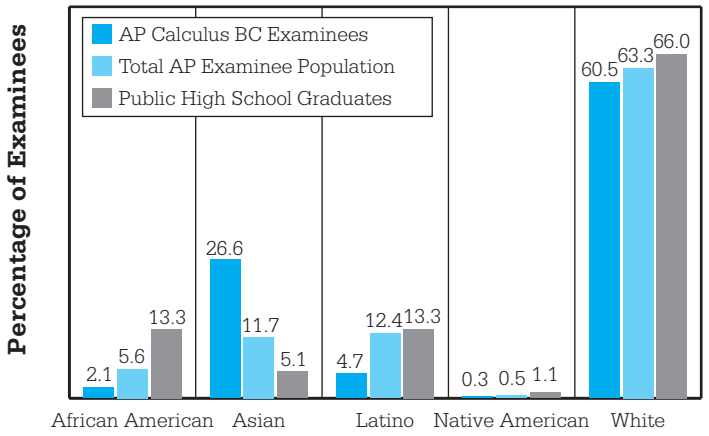


Figure 4: AP Calculus BC Examinees by Race and Ethnicity, 2005



AP Credit Policy Info on the Web

Information about AP credit and placement policies at more than 1,000 colleges and universities is available on the College Board's Web site at **www.collegeboard.com/ap/creditpolicy.**

The AP Calculus Courses

The two AP Calculus courses are each designed to be the equivalent of a first-year, college-level course in calculus. Calculus AB and Calculus BC are primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important. Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. The focus of the courses is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Although a facility with manipulation and computational competence is an important outcome, it is not the core of these courses. Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics.

The Development Committee creates the guidelines for the AP Calculus courses and designs the AP Exams. Periodically the Development Committee conducts curriculum surveys, sent to instructors who teach the comparable college-level course, that help ensure that the AP Calculus courses remain current with concepts and themes as taught in college and university classrooms.

The Development Committee has created goals for students and a topic outline that covers the main subject areas that should be taught for each course.

The goals for both Calculus courses are:

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and should be able to use integrals to solve a variety of problems.
- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.
- Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Topic Outline for Calculus (**topics listed in bold apply only to Calculus BC**):

1. Functions, Graphs, and Limits
   a. Analysis of graphs
   b. Limits of functions (including one-sided limits)
   c. Asymptotic and unbounded behavior
   d. Continuity as a property of functions
   e. **Parametric, polar, and vector functions**
2. Derivatives
   a. Concept of the derivative
   b. Derivative at a point
   c. Derivative as a function
   d. Second derivatives
   e. Applications of derivatives
   f. Computation of derivatives
3. Integrals
   a. Interpretations and properties of definite integrals
   b. Applications of integrals
   c. Fundamental Theorem of Calculus
   d. Techniques of antidifferentiation
   e. Applications of antidifferentiation
   f. Numerical approximations to definite integrals
4. **Polynomial Approximations and Series**
   a. **Concept of a series**
   b. **Series of constants**
   c. **Taylor series**

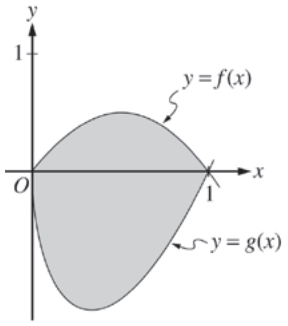
Beginning in fall 2006, AP Calculus teachers and principals of schools where AP Calculus is taught must certify that their 2007-08 courses follow all the requirements stipulated by the Development Committee, including using a college-level textbook, in order to ensure that the AP course reflects college-level standards. By completing this AP Course Audit, high schools will receive individual licenses to label their calculus courses "AP." In fall 2007, colleges and universities will receive a list of all high schools authorized to use the "AP" designation for their calculus courses.

The AP Calculus Exams

Both AP Calculus Exams include a multiple-choice section that tests proficiency in a wide variety of topics and a free-response section that requires the student to demonstrate the ability to solve problems involving a more extended chain of reasoning. Both the multiple-choice and free-response sections contain parts where a graphing calculator is required and parts where calculator use is prohibited. Equal weight is placed on the multiple-choice and free-response sections in determining the final score. The time allotted for each AP Calculus Exam is 3 hours and 15 minutes. A Calculus AB subscore grade is reported based on performance on the portion of the Calculus BC exam devoted to Calculus AB topics (approximately 60 percent of the exam). This subscore is designed to provide more information about the student to the mathematics departments at colleges and universities. It is recommended that the same credit and placement policies that apply to the Calculus AB exam also be applied to the Calculus AB subscore of the Calculus BC exam, as common topics are tested at the same conceptual level in both exams.

AP Calculus free-response questions from recent exam years are listed below.

Question 1 (Calculus AB and BC)



Let f and g be the functions given by $f(x)=2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1 - x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

Question 2 (Calculus AB and BC)

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5 p.m. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5 p.m. ($t = 17$). After 5 p.m., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

Question 3 (Calculus AB and BC)

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$ in degrees Celsius (°C) of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$ and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

Question 4 (Calculus BC)

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \ge 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$.

The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position (1, 8).

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

Question 5 (Calculus BC)

The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

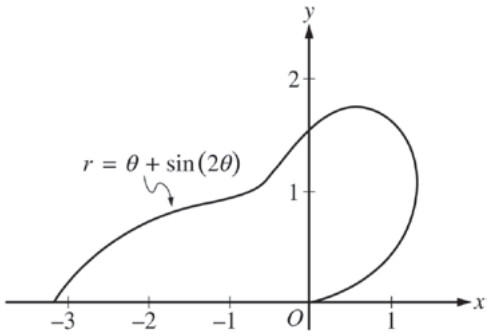
for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

(c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

Question 6 (Calculus BC)



The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \leq \theta \leq \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \leq \theta \leq \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

How to Get Involved

There are many ways college and university faculty members can help maintain the high standards of the AP Program:

- Participate in a college comparability study
- Be an AP Reader
- Contribute multiple-choice test items for the AP Exam
- Become an AP Faculty Consultant

For more information, please go to: **apcentral.collegeboard.com/highered/getinvolved**

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The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 5,000 schools, colleges, universities, and other educational organizations. Each year, the College Board serves seven million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns. For further information, visit www.collegeboard.com.

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