



## **AP<sup>®</sup> Calculus AB 2012 Free-Response Questions**

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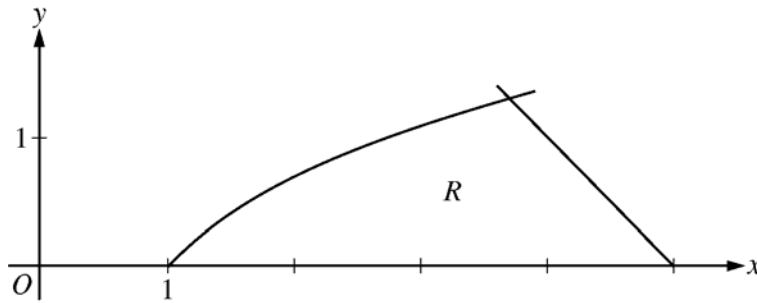
**2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS****CALCULUS AB  
SECTION II, Part A  
Time—30 minutes  
Number of problems—2**

**A graphing calculator is required for these problems.**

$t$ (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.
- (a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \leq t \leq 25$ , the function  $W$  that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t} \cos(0.06t)$ . Based on the model, what is the temperature of the water at time  $t = 25$ ?
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2. Let  $R$  be the region in the first quadrant bounded by the  $x$ -axis and the graphs of  $y = \ln x$  and  $y = 5 - x$ , as shown in the figure above.
- (a) Find the area of  $R$ .
- (b) Region  $R$  is the base of a solid. For the solid, each cross section perpendicular to the  $x$ -axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.
- (c) The horizontal line  $y = k$  divides  $R$  into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of  $k$ .
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END OF PART A OF SECTION II

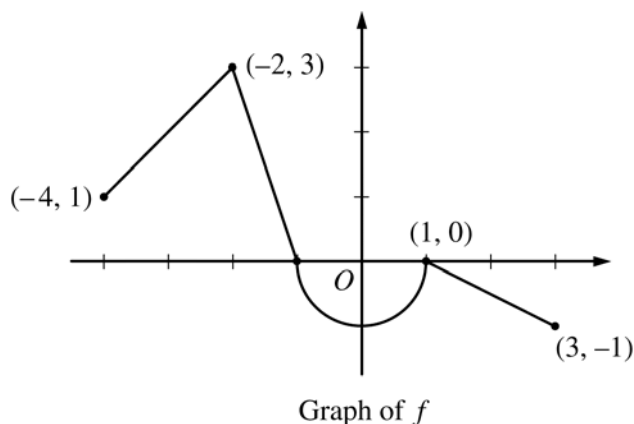
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CALCULUS AB  
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



Graph of  $f$

3. Let  $f$  be the continuous function defined on  $[-4, 3]$  whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let  $g$  be the function given by  $g(x) = \int_1^x f(t) dt$ .
- (a) Find the values of  $g(2)$  and  $g(-2)$ .
- (b) For each of  $g'(-3)$  and  $g''(-3)$ , find the value or state that it does not exist.
- (c) Find the  $x$ -coordinate of each point at which the graph of  $g$  has a horizontal tangent line. For each of these points, determine whether  $g$  has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- (d) For  $-4 < x < 3$ , find all values of  $x$  for which the graph of  $g$  has a point of inflection. Explain your reasoning.
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4. The function  $f$  is defined by  $f(x) = \sqrt{25 - x^2}$  for  $-5 \leq x \leq 5$ .

(a) Find  $f'(x)$ .

(b) Write an equation for the line tangent to the graph of  $f$  at  $x = -3$ .

(c) Let  $g$  be the function defined by  $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

Is  $g$  continuous at  $x = -3$ ? Use the definition of continuity to explain your answer.

(d) Find the value of  $\int_0^5 x\sqrt{25 - x^2} dx$ .

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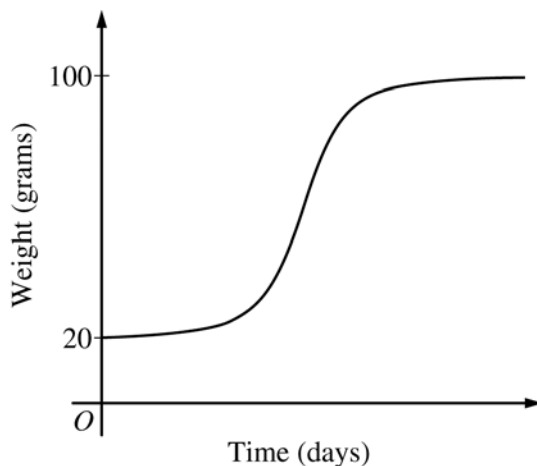
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5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.



- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .

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6. For  $0 \leq t \leq 12$ , a particle moves along the  $x$ -axis. The velocity of the particle at time  $t$  is given by  $v(t) = \cos\left(\frac{\pi}{6}t\right)$ . The particle is at position  $x = -2$  at time  $t = 0$ .
- (a) For  $0 \leq t \leq 12$ , when is the particle moving to the left?
  - (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time  $t = 0$  to time  $t = 6$ .
  - (c) Find the acceleration of the particle at time  $t$ . Is the speed of the particle increasing, decreasing, or neither at time  $t = 4$ ? Explain your reasoning.
  - (d) Find the position of the particle at time  $t = 4$ .
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**STOP**

**END OF EXAM**