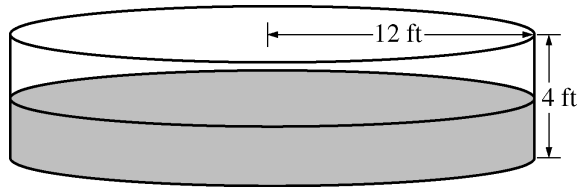


**AP<sup>®</sup> CALCULUS AB**  
**2010 SCORING GUIDELINES (Form B)**

**Question 3**

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, water is pumped into the pool at the rate  $P(t)$  cubic feet per hour. The table above gives values of  $P(t)$  for selected values of  $t$ . During the same time interval, water is leaking from the pool at the rate  $R(t)$  cubic feet per hour, where  $R(t) = 25e^{-0.05t}$ . (Note: The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is given by  $V = \pi r^2 h$ .)

- (a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval  $0 \leq t \leq 12$  hours. Show the computations that lead to your answer.
- (b) Calculate the total amount of water that leaked out of the pool during the time interval  $0 \leq t \leq 12$  hours.
- (c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time  $t = 12$  hours. Round your answer to the nearest cubic foot.
- (d) Find the rate at which the volume of water in the pool is increasing at time  $t = 8$  hours. How fast is the water level in the pool rising at  $t = 8$  hours? Indicate units of measure in both answers.

$$(a) \int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$$

$$(b) \int_0^{12} R(t) dt = 225.594 \text{ ft}^3$$

$$(c) 1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$$

At time  $t = 12$  hours, the volume of water in the pool is approximately  $1434 \text{ ft}^3$ .

$$(d) V'(t) = P(t) - R(t)$$

$$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$$

$$V = \pi(12)^2 h$$

$$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$$

$$\left. \frac{dh}{dt} \right|_{t=8} = \frac{1}{144\pi} \cdot \left. \frac{dV}{dt} \right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$$

$$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

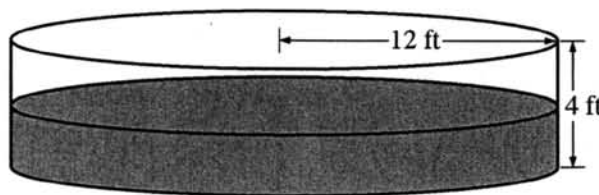
$$1 : \text{answer}$$

$$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left. \frac{dh}{dt} \right|_{t=8} \\ 1 : \text{units of } \text{ft}^3/\text{hr} \text{ and } \text{ft/hr} \end{cases}$$

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3A,

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

$$\text{WATER ADDED INTO POOL} = \int_0^{12} P(t) dt \approx 4(46 + 57 + 62) = \boxed{660 \text{ ft}^3}$$

ABOUT 660  $\text{ft}^3$  OF WATER ARE ADDED TO THE POOL FROM  $t = 0 \text{ h}$  TO  $t = 12 \text{ h}$

Work for problem 3(b)

$$\text{WATER LEAVED} = \int_0^{12} R(t) dt = \int_0^{12} (25e^{-.05t}) dt = \boxed{225.594 \text{ ft}^3}$$

225.594  $\text{ft}^3$  OF WATER LEAVES FROM THE POOL FROM  $t = 0 \text{ h}$  TO  $t = 12 \text{ h}$

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Continue problem 3 on page 9.

Work for problem 3(c)

$$V = (\text{INITIAL WATER}) + (\text{WATER IN}) - (\text{WATER OUT}) = 1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1660 - 225.594 =$$

$$\rightarrow = 1434.406 \text{ ft}^3$$

THE VOLUME OF WATER IN THE POOL AT TIME  $t = 12 \text{ h}$  IS ABOUT 1434 ft<sup>3</sup>

$$1434 \text{ ft}^3$$

Work for problem 3(d)

$$\frac{dV}{dt} = (\text{RATE WATER IN}) - (\text{RATE WATER OUT})$$

$$\frac{dV}{dt} = P(8) - R(8) = 43.242 \text{ ft}^3/\text{hour}$$

AT  $t = 8 \text{ h}$ , THE VOLUME IN THE TANK IS INCREASING AT 43.242 ft<sup>3</sup>/hour

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$43.242 = 452389 \frac{dh}{dt}$$

$$\frac{dh}{dt} = .096 \text{ ft}/\text{hour}$$

AT  $t = 8 \text{ h}$ , THE WATER LEVEL IS RISING AT .096 ft/hour

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

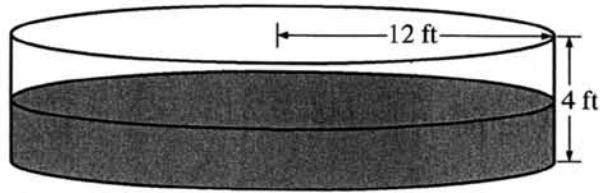
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3B

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

Midpoints are  $x=2, 6, 10$ ,  $p(t)=46, 57, 62$ .  
 $Sum = 4(46+57+62) = 660$ .

Work for problem 3(b)

$$V_{\text{water leaking out}} = \int_0^{12} R(t) \cdot dt = \int_0^{12} 25e^{-0.05t} \cdot dt$$

$$= 225.594$$

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Continue problem 3 on page 9.

Work for problem 3(c)

$$\begin{aligned}
 V &= V_{\text{pumped}} - V_{\text{leak}} \\
 &= 660 - 225.6 = 434.4 \text{ cubic feet} \\
 &\approx 434 \text{ cubic feet}
 \end{aligned}$$

Work for problem 3(d)

$$\begin{aligned}
 \frac{d}{dt} [p(t) - R(t)] &= \frac{d}{dt} (80 - 25e^{-0.05t}) \\
 &= -25 \cdot (-0.05) e^{-0.05t}
 \end{aligned}$$

$$\begin{aligned}
 \text{since } t=8 &\rightarrow +25 \times 0.05 \times 8 \times e^{-0.05 \times 8} \\
 &= 2.467
 \end{aligned}$$

Thus, the volume of water is increasing at the rate of 2.467  $\text{cu ft}^3/\text{h}$  at  $t=8$

$$\begin{aligned}
 V &= \pi r^2 h \\
 \frac{dV}{dt} &= \pi r^2 \frac{dh}{dt} \\
 \frac{dh}{dt} &= \frac{2.467}{\pi \cdot 12^2} = 0.0055 \text{ ft/h}
 \end{aligned}$$

The water level is rising at the rate of 0.0055 ft/h at  $t=8$ .

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

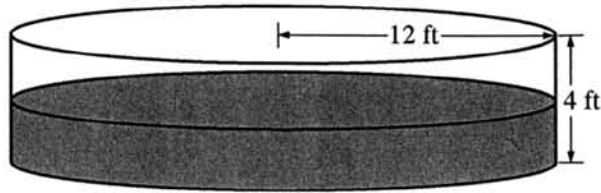
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3C

$t$	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



Work for problem 3(a)

The approximate total amount of water

$$= \frac{(0+53) \times 4}{2} + \frac{(53+60) \times 4}{2} + \frac{(60+63) \times 4}{2}$$

$$= 289 \times 2$$

$$= 578 \text{ cubic feet}$$

Work for problem 3(b)

The total amount of water leaking out

$$= \int_0^{12} 25e^{-0.05t} dt = 255.594 \text{ cubic feet}$$

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Continue problem 3 on page 9.

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3C<sub>2</sub>

Work for problem 3(c)

$$\begin{aligned} &\text{The approximate volume of water in time } t=12 \\ &= 1000 + 578 - 255.594 \\ &= 1322 \text{ cubic feet} \end{aligned}$$

Work for problem 3(d)

$$\begin{aligned} \text{The rate} &= \frac{d}{dt} (60 - 25e^{-0.01t}) = 1.25 \cdot e^{-0.01t} \\ &= 0.8379 \text{ cubic feet per second} \end{aligned}$$

$$\begin{aligned} \text{The rate of rising} &= \frac{d}{dt} (1.25 \cdot e^{-0.01t}) \\ &= -0.0625 \cdot e^{-0.01t} \\ &= -0.419 \text{ cubic feet per square second} \end{aligned}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON  
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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**2010 SCORING COMMENTARY (Form B)**

**Question 3**

**Sample: 3A**

**Score: 9**

The student earned all 9 points.

**Sample: 3B**

**Score: 6**

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student does not use the initial condition, and the point was not earned. In part (d) the student's presented value for  $V'(8)$  is incorrect. The relationship between  $\frac{dV}{dt}$  and  $\frac{dh}{dt}$  is correct, and the value of  $\frac{dh}{dt}$  is consistent with the student's  $V'(8)$ . The second and third points were earned. The units on  $\frac{dh}{dt}$  are incorrect.

**Sample: 3C**

**Score: 3**

The student earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not use a midpoint Riemann sum. In part (b) the student's work is correct. In part (c) the student correctly combines the results from parts (a) and (b) along with the initial condition. In part (d) the student's work is incorrect.