

AP[®] STATISTICS
2009 SCORING GUIDELINES (Form B)

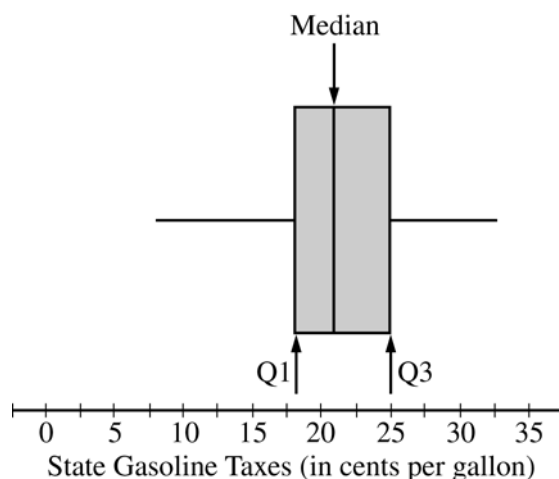
Question 1

Intent of Question

The primary goals of this question were to assess students' ability to (1) approximate the median and the IQR of a distribution from the boxplot and (2) recalibrate the values of the median and the IQR of the distribution if the same constant value is added to each observation in the distribution.

Solution

Part (a):



The median and quartiles are marked and labeled on the boxplot above. The median is approximately 21 cents per gallon.

The first and third quartiles are approximately 18 cents per gallon and 25 cents per gallon, respectively. The IQR is $Q3 - Q1$, which is approximately $25 - 18 = 7$ cents per gallon.

Part (b):

After adding 18.4 cents per gallon to each of the state taxes, the median of the combined gasoline taxes would be the median of the state tax plus the federal tax, which is approximately $21 + 18.4 = 39.4$ cents per gallon.

Although the quartiles of the combined gasoline taxes will change ($Q1 = 18 + 18.4 = 36.4$ cents per gallon and $Q3 = 25 + 18.4 = 43.4$ cents per gallon), the IQR will remain the same as it was for the state taxes at 7 cents per gallon ($43.4 - 36.4 = 7$).

AP[®] STATISTICS
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Question 1 (continued)

Scoring

Parts (a) and (b) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the student identifies reasonable values for the median and IQR and justifies them by marking and labeling the boxplot.

Partially correct (P) if the student identifies reasonable values for the median and IQR but does not mark or label the boxplot *OR* identifies, marks, and labels only one value (median or IQR).

Incorrect (I) if the student identifies neither value *OR* identifies only one value but fails to mark and label the boxplot.

Part (b) is scored as follows:

Essentially correct (E) if the student gives a median that is 18.4 cents per gallon larger than the median identified in part (a), gives an IQR that is the same single number found in part (a), *AND* provides a reasonable justification for at least one of these values.

Partially correct (P) if the student provides only one correct value (either the median or the IQR) *AND* provides a justification.

Incorrect (I) if the student gives incorrect values for the median and IQR *OR* provides only one correct value with no justification.

4 Complete Response

Both parts essentially correct

3 Substantial Response

One part essentially correct and one part partially correct

2 Developing Response

One part essentially correct and one part incorrect

OR

Both parts partially correct

1 Minimal Response

One part partially correct and one part incorrect

STATISTICS

SECTION II

Part A

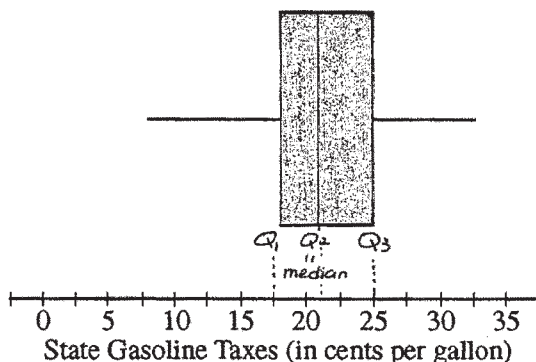
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. As gasoline prices have increased in recent years, many drivers have expressed concern about the taxes they pay on gasoline for their cars. In the United States, gasoline taxes are imposed by both the federal government and by individual states. The boxplot below shows the distribution of the state gasoline taxes, in cents per gallon, for all 50 states on January 1, 2006.



- (a) Based on the boxplot, what are the approximate values of the median and the interquartile range of the distribution of state gasoline taxes, in cents per gallon? Mark and label the boxplot to indicate how you found the approximated values.

The median of the distribution of state gasoline taxes is about 21.5 cents per gallon; the interquartile range of the distribution is about 7.5 cents per gallon.

The median is equal to Q_2 , which is 50th percentile, and can be found on the boxplot. The interquartile range can be found by subtracting the value of Q_1 from Q_3 . ($25 - 17.5 = 7.5$). Q_1 is 25th percentile and Q_3 is 75th percentile.

GO ON TO THE NEXT PAGE.

- (b) The federal tax imposed on gasoline was 18.4 cents per gallon at the time the state taxes were in effect. The federal gasoline tax was added to the state gasoline tax for each state to create a new distribution of combined gasoline taxes. What are approximate values, in cents per gallon, of the median and interquartile range of the new distribution of combined gasoline taxes? Justify your answer.

The new median is 39.9 cents per gallon ($21.5 + 18.4 = 39.9$), and the new interquartile range remains at 7.5 cents per gallon. The boxplot of combined gasoline taxes shifts to the right of the boxplot of state gasoline taxes by 18.4 (the federal gasoline tax). ~~However,~~ Therefore, 18.4 should be added to the old median to find the new median. The interquartile range is unchanged because both Q_1 and Q_3 increase by the same amount (18.4 cents per gallon) - leaving the value of $(Q_3 - Q_1)$ unchanged.

GO ON TO THE NEXT PAGE.

STATISTICS

SECTION II

Part A

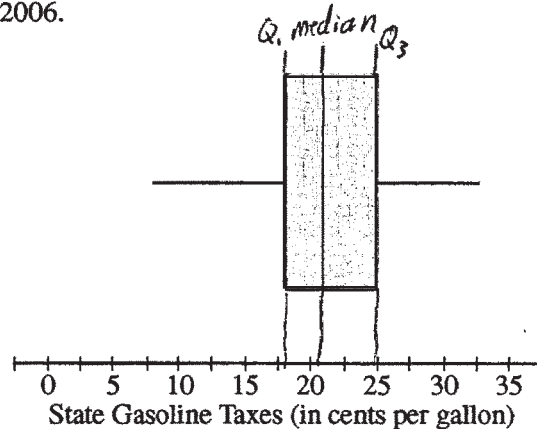
Questions 1-5

Spend about 65 minutes on this part of the exam.

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1. As gasoline prices have increased in recent years, many drivers have expressed concern about the taxes they pay on gasoline for their cars. In the United States, gasoline taxes are imposed by both the federal government and by individual states. The boxplot below shows the distribution of the state gasoline taxes, in cents per gallon, for all 50 states on January 1, 2006.



- (a) Based on the boxplot, what are the approximate values of the median and the interquartile range of the distribution of state gasoline taxes, in cents per gallon? Mark and label the boxplot to indicate how you found the approximated values.

$$\text{median} \approx 22 \text{ ¢/gal}$$

$$\text{IQR} = Q_3 - Q_1$$

$$\approx 25 - 16$$

$$= 9 \text{ ¢/gallon}$$

GO ON TO THE NEXT PAGE.

- (b) The federal tax imposed on gasoline was 18.4 cents per gallon at the time the state taxes were in effect. The federal gasoline tax was added to the state gasoline tax for each state to create a new distribution of combined gasoline taxes. What are approximate values, in cents per gallon, of the median and interquartile range of the new distribution of combined gasoline taxes? Justify your answer.

The additional federal tax of 18.4 ¢/gal added on to the individual state gasoline tax would shift all values of gasoline prices up by 18.4 ¢/gal, including the median, Q_1 , and Q_3 .

$$\begin{aligned}\text{Hence, median} &\approx 22 + 18.4 \\ &= 40.4 \text{ ¢/gal}\end{aligned}$$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= (25 + 18.4) - (16 + 18.4) \\ &= 9 \text{ ¢/gal}\end{aligned}$$

~~the~~

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STATISTICS
SECTION II
Part A

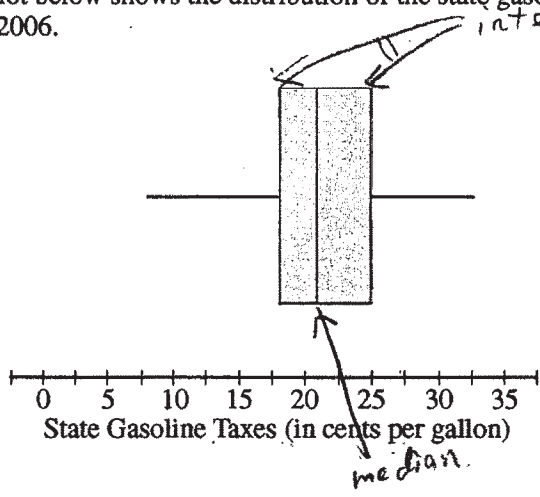
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- 1. As gasoline prices have increased in recent years, many drivers have expressed concern about the taxes they pay on gasoline for their cars. In the United States, gasoline taxes are imposed by both the federal government and by individual states. The boxplot below shows the distribution of the state gasoline taxes, in cents per gallon, for all 50 states on January 1, 2006.



- (a) Based on the boxplot, what are the approximate values of the median and the interquartile range of the distribution of state gasoline taxes, in cents per gallon? Mark and label the boxplot to indicate how you found the approximated values.

interquartile : ~~7.5~~ ~~7.5~~ ~~17.5~~ 17.5 - 24.5
 median : 22

GO ON TO THE NEXT PAGE.

- (b) The federal tax imposed on gasoline was 18.4 cents per gallon at the time the state taxes were in effect. The federal gasoline tax was added to the state gasoline tax for each state to create a new distribution of combined gasoline taxes. What are approximate values, in cents per gallon, of the median and interquartile range of the new distribution of combined gasoline taxes? Justify your answer.

New interquartile: ~~29.5~~ 35.9 - 42.9

New median: 40.6

Because gasoline taxes are imposed by both government & by individual states

$$\text{Tax}_{\text{tot}} = \text{Tax}_{\text{gov}} + \text{Tax}_{\text{states}}$$

$$\text{So } 29.5 + 18.4 = \text{42.9}$$

$$17.5 + 18.4 = 35.9$$

$$22 + 18.4 = 40.4$$

GO ON TO THE NEXT PAGE.

AP[®] STATISTICS
2009 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A

Score: 4

Each question in the two parts is correctly and clearly answered. In part (a) the median and quartiles are marked and labeled on the boxplot. Approximated values are given for the median and interquartile range, with the computation shown for the interquartile range. The units of cents per gallon are included for all values. Thus part (a) was scored as essentially correct. In part (b) a clear explanation is provided of how the addition of 18.4 cents per gallon to each individual state tax shifts the entire distribution 18.4 units to the right, so the median increases by 18.4 to become “39.9 cents per gallon” while the interquartile range remains “unchanged” at “7.5 cents per gallon.” Thus part (b) was scored as essentially correct. Because part (a) and part (b) were both essentially correct, this complete response received a score of 4.

Sample: 1B

Score: 3

Each question in the two parts is correctly and clearly answered, except that in part (a) an incorrect value is given for the first quartile. The tick mark between 15 and 20 on the horizontal scale represents 17.5, not 16, so part (a) was scored as partially correct. In part (b) the justification of how to find the interquartile range of the distribution of the combined gasoline taxes includes a correct, but optional, computation that verifies that the interquartile range remains unchanged. Part (b) was scored as essentially correct. Because part (a) was partially correct and part (b) was essentially correct, this substantial response received a score of 3.

Sample: 1C

Score: 2

All parts of both questions are answered succinctly in this response, but it includes a significant error. In part (a) the interquartile range is given as “17.5 – 26.5,” rather than as the correct single value of approximately 7. Thus in part (b), while the quartiles for the combined gasoline taxes are correctly computed, the conclusion that the interquartile range does not change when the same federal tax per gallon is added to each state tax cannot be reached. So both parts (a) and (b) were scored as partially correct. This developing response received a score of 2.