



AP[®] Calculus BC 2007 Scoring Guidelines Form B

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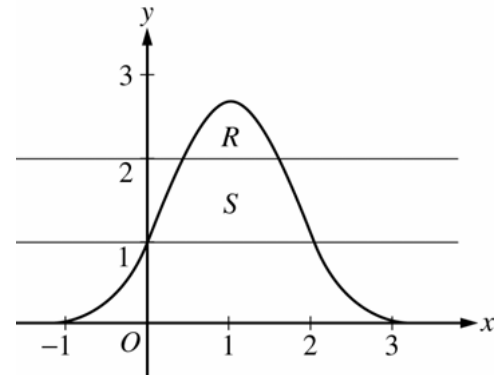
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Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.



- (a) Find the area of R .
 (b) Find the area of S .
 (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$
 Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q (e^{2x-x^2} - 2) dx = 0.514$

3 : { 1 : integrand
 1 : limits
 1 : answer

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 (e^{2x-x^2} - 1) dx - \text{Area of } R$
 $= 2.06016 - \text{Area of } R = 1.546$

3 : { 1 : integrand
 1 : limits
 1 : answer

OR

$\int_0^P (e^{2x-x^2} - 1) dx + (Q - P) \cdot 1 + \int_Q^2 (e^{2x-x^2} - 1) dx$
 $= 0.219064 + 1.107886 + 0.219064 = 1.546$

(c) Volume $= \pi \int_P^Q \left((e^{2x-x^2} - 1)^2 - (2 - 1)^2 \right) dx$

3 : { 2 : integrand
 1 : constant and limits

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Question 2

An object moving along a curve in the xy -plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln(t^2 + 1)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1}x = \arctan x$)

- (a) Find the speed of the object at time $t = 4$.
 (b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.
 (c) Find $x(4)$.
 (d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

(a) Speed = $\sqrt{x'(4)^2 + y'(4)^2} = 2.912$

1 : speed at $t = 4$

(b) Distance = $\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = 6.423$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $x(4) = x(0) + \int_0^4 x'(t) dt$
 $= -3 + 2.10794 = -0.892$

3 : $\begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = -3 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The slope is 2, so $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln(t^2 + 1) = 2 \arctan\left(\frac{t}{1+t}\right)$.

3 : $\begin{cases} 1 : \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1 : t\text{-value} \\ 1 : \text{values for } x'' \text{ and } y'' \end{cases}$

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is
 $\langle x''(t), y''(t) \rangle|_{t=1.35766} = \langle 0.135, 0.955 \rangle$.

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Question 3

The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.

- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at $0.286^{\circ}\text{F}/\text{mph}$.

(b) The average rate of change of W over the interval $5 \leq v \leq 60$ is $\frac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254 .
 $W'(v) = \frac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

(c) $\left. \frac{dW}{dt} \right|_{t=3} = \left(\frac{dW}{dv} \cdot \frac{dv}{dt} \right) \Big|_{t=3} = W'(35) \cdot 5 = -0.892^{\circ}\text{F}/\text{hr}$

OR

$$W = 55.6 - 22.1(20 + 5t)^{0.16}$$

$$\left. \frac{dW}{dt} \right|_{t=3} = -0.892^{\circ}\text{F}/\text{hr}$$

Units of $^{\circ}\text{F}/\text{mph}$ in (a) and $^{\circ}\text{F}/\text{hr}$ in (c)

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$$

$$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$$

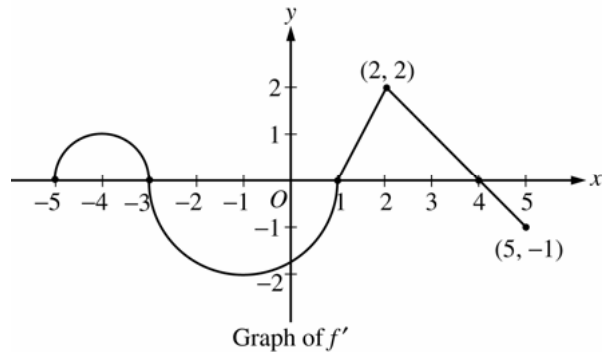
$$3 : \begin{cases} 1 : \frac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \text{or} \\ \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$$

1 : units in (a) and (c)

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Question 4

Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.



- (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3, 1, 4$
 f' changes from positive to negative at -3 and 4 .
 Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : $\left\{ \begin{array}{l} 1 : x\text{-values} \\ 1 : \text{justification} \end{array} \right.$

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4, -1$, and 2 . Thus, the graph of f has points of inflection when $x = -4, -1$, and 2 .

2 : $\left\{ \begin{array}{l} 1 : x\text{-values} \\ 1 : \text{justification} \end{array} \right.$

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : $\left\{ \begin{array}{l} 1 : \text{intervals} \\ 1 : \text{explanation} \end{array} \right.$

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

3 : $\left\{ \begin{array}{l} 1 : \text{identifies } x = 1 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{value and explanation} \end{array} \right.$

$$f(-5) = 3 + \int_1^{-5} f'(x) dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^5 f'(x) dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

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Question 5

Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

- (a) Find $\frac{d^2y}{dx^2}$ in terms of x and y .
- (b) Find the values of the constants m , b , and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.
- (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.
- (d) Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k .

(a) $\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5$

2 : $\begin{cases} 1 : 3 + 2\frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$

(b) If $y = mx + b + e^{rx}$ is a solution, then
 $m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1$.

3 : $\begin{cases} 1 : \frac{dy}{dx} = m + re^{rx} \\ 1 : \text{value for } r \\ 1 : \text{values for } m \text{ and } b \end{cases}$

If $r \neq 0$: $m = 2b + 1$, $r = 2$, $0 = 3 + 2m$,
 so $m = -\frac{3}{2}$, $r = 2$, and $b = -\frac{5}{4}$.

OR

If $r = 0$: $m = 2b + 3$, $r = 0$, $0 = 3 + 2m$,
 so $m = -\frac{3}{2}$, $r = 0$, $b = -\frac{9}{4}$.

(c) $f\left(\frac{1}{2}\right) \approx f(0) + f'(0) \cdot \frac{1}{2} = -2 + (-3) \cdot \frac{1}{2} = -\frac{7}{2}$
 $f'\left(\frac{1}{2}\right) \approx 3\left(\frac{1}{2}\right) + 2\left(-\frac{7}{2}\right) + 1 = -\frac{9}{2}$
 $f(1) \approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) \cdot \frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right) \cdot \frac{1}{2} = -\frac{23}{4}$

2 : $\begin{cases} 1 : \text{Euler's method with 2 steps} \\ 1 : \text{Euler's approximation for } f(1) \end{cases}$

(d) $g'(0) = 3 \cdot 0 + 2 \cdot k + 1 = 2k + 1$
 $g(1) \approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0$
 $k = -\frac{1}{3}$

2 : $\begin{cases} 1 : g(0) + g'(0) \cdot 1 \\ 1 : \text{value of } k \end{cases}$

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Question 6

Let f be the function given by $f(x) = 6e^{-x/3}$ for all x .

- (a) Find the first four nonzero terms and the general term for the Taylor series for f about $x = 0$.
- (b) Let g be the function given by $g(x) = \int_0^x f(t) dt$. Find the first four nonzero terms and the general term for the Taylor series for g about $x = 0$.
- (c) The function h satisfies $h(x) = kf'(ax)$ for all x , where a and k are constants. The Taylor series for h about $x = 0$ is given by

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

Find the values of a and k .

(a)
$$f(x) = 6 \left[1 - \frac{x}{3} + \frac{x^2}{2!3^2} - \frac{x^3}{3!3^3} + \cdots + \frac{(-1)^n x^n}{n!3^n} + \cdots \right]$$

$$= 6 - 2x + \frac{x^2}{3} - \frac{x^3}{27} + \cdots + \frac{6(-1)^n x^n}{n!3^n} + \cdots$$

3 : $\left\{ \begin{array}{l} 1 : \text{two of } 6, -2x, \frac{x^2}{3}, -\frac{x^3}{27} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \\ \langle -1 \rangle \text{ missing factor of } 6 \end{array} \right.$

(b) $g(0) = 0$ and $g'(x) = f(x)$, so

$$g(x) = 6 \left[x - \frac{x^2}{6} + \frac{x^3}{3!3^2} - \frac{x^4}{4!3^3} + \cdots + \frac{(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots \right]$$

$$= 6x - x^2 + \frac{x^3}{9} - \frac{x^4}{4(27)} + \cdots + \frac{6(-1)^n x^{n+1}}{(n+1)!3^n} + \cdots$$

3 : $\left\{ \begin{array}{l} 1 : \text{two terms} \\ 1 : \text{remaining terms} \\ 1 : \text{general term} \\ \langle -1 \rangle \text{ missing factor of } 6 \end{array} \right.$

(c) $f'(x) = -2e^{-x/3}$, so $h(x) = -2ke^{-ax/3}$

$$h(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = e^x$$

$$-2ke^{-ax/3} = e^x$$

$$\frac{-a}{3} = 1 \text{ and } -2k = 1$$

3 : $\left\{ \begin{array}{l} 1 : \text{computes } kf'(ax) \\ 1 : \text{recognizes } h(x) = e^x, \\ \text{or} \\ \text{equates 2 series for } h(x) \\ 1 : \text{values for } a \text{ and } k \end{array} \right.$

$$a = -3 \text{ and } k = -\frac{1}{2}$$

OR

$$f'(x) = -2 + \frac{2}{3}x + \cdots, \text{ so}$$

$$h(x) = kf'(ax) = -2k + \frac{2}{3}akx + \cdots$$

$$h(x) = 1 + x + \cdots$$

$$-2k = 1 \text{ and } \frac{2}{3}ak = 1$$

$$k = -\frac{1}{2} \text{ and } a = -3$$