LAB ONE

Diluting Gravity: A Computer-Based Laboratory

Introduction

The kinematics of one-dimensional motion is usually the first material covered in an introductory physics course. When confined to the case of constant acceleration, one-dimensional kinematics provides a subject that is amenable to classroom analysis and laboratory study both in non-calculus and calculus-based courses. The kinematic equations describe the position x and the velocity u of an object in terms of its initial position x_0, its initial velocity u_0, the time t, and the object's acceleration a. The two equations used in this lab are

$$x = x_0 + u_0 t + \frac{1}{2} a t^2 \qquad (1\text{-}1)$$

$$u = u_0 + at \qquad (1\text{-}2)$$

If the object of interest is falling freely near Earth's surface, the magnitude of the acceleration is commonly denoted by g, which has a magnitude of 9.8 m/s^2. In this lab, a local value of g is measured. We dilute g by having the object accelerate down an inclined plane. We reduce the friction that the object experiences by using a glider on an air track as the object. When we raise one end of the air track so that it makes an angle q with respect to the horizontal, the acceleration down the inclined air track is given by the equation

$$a = g \sin q \qquad (1\text{-}3)$$

If we start the glider from rest, Equations 1-1 and 1-2 may now be written as

$$x = x_0 + \frac{1}{2}(g \sin q)\, t^2 \qquad (1\text{-}4)$$

$$u = (g \sin q)\, t \qquad (1\text{-}5)$$

Equipment

Two ways of doing this lab are presented here. Both require an **air track** and **air track glider.** One method uses **a pair of photogates** to measure the time during which the object moves on the track. The other method uses a **sonic ranger** and **computer data acquisition system** to obtain the glider's position as a function of time. If time permits, it is useful to do the lab both ways in order to introduce students to the advantages and disadvantages of computer data acquisition.

Diluting Gravity 1

Experimental Procedure

The students find the factor $\sin q$ by taking the ratio of the height *H* to which they raise one end of the air track to the distance *L* between the air track supports, as shown in Figure 1.1. An easy and reproducible way to raise the track is to place a metal block under the supports at one end of the air track. A pair of blocks, one 0.5 inch high and one 1.0 inch high, allow the students to obtain data at three different track inclinations. The height of the metal blocks may be

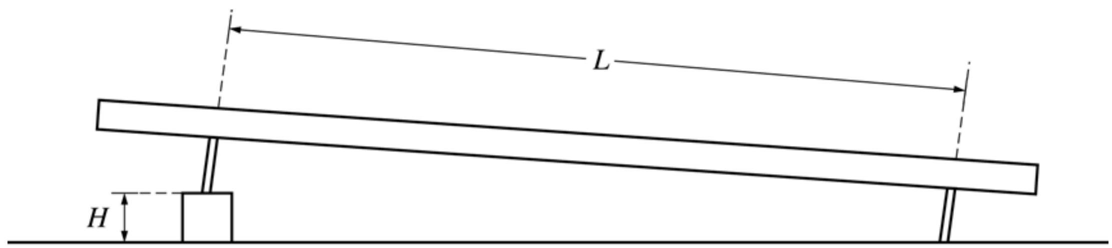

Figure 1.1

measured with a vernier caliper, and the distance between the air track supports with a meter stick. When the students calculate $\sin q$, they need to be careful about the number of significant figures they use. For example, suppose the distance *L* between the supports is 90.5 ± 0.1 cm, and the height *H* of the metal block is 2.531 ± 0.001 cm. The three significant figures in the measurement of *L* mean that the calculated value of $\sin q$ should have only three significant figures.

The time measurement is taken by the two photogates, which are separated by a distance *S*. The timer starts when the glider passes through the first gate, and stops when the glider passes through the second. The students vary the photogate separation *S*, and record the time it takes for the glider to traverse those separations. If the glider is released so that it triggers the first photogate as soon as the glider starts down the ramp, then Equation 1-4 may be written as

$$S = \frac{1}{2}(g \sin q)t^2 \qquad \text{(1-6)}$$

Thus a plot of t^2 versus *S* should yield a straight line with a slope equal to the reciprocal of $\frac{1}{2} g \sin q$. This may be used to calculate a local value of *g*.

Error Analysis

The two places where problems arise in this lab are the error introduced because the glider is already moving when it triggers the first photogate, and measuring the separation of the photogates. Determining the error associated with measuring the separation of the gates is straightforward. The error associated with the delayed triggering of the first photogate is a bit more involved. If, after being released, the glider travels a distance *R* before it triggers the first photogate, it has a nonzero velocity at the beginning of the interval *S*. Now we need to determine that initial non-zero velocity, to determine the size of the error it introduces in the experimental value of *g*. Rearranging Equations 1-1 and 1-2 to eliminate time, we obtain

$$2a(x - x_0) = v_1^2 - v_0^2 \qquad (1\text{-}7)$$

Here v_1 is the velocity as the glider reaches the first photogate, the initial velocity v_0 is zero, the acceleration *a* is equal to $g \sin\theta$, and $(x - x_0)$ is equal to *R*. The velocity v_1 is then given by

$$v_1^2 = 2gR \sin\theta \qquad (1\text{-}8)$$

Including this non-zero velocity in the equation for the distance between the photogates gives

$$S = v_1 t + \frac{1}{2}(g \sin\theta) t^2 \qquad (1\text{-}9)$$

Now assume that we want the error in the experimental value of *g* to be less than 5% due to the nonzero velocity at the first photogate. We need to determine the maximum value of *R* that will ensure this accuracy. Solving for *g* in both Equation 1-6 and 1-9, we can then take the difference of these two experimental values of *g* and set that equal to 5% of the accepted value of *g*. This gives us the relationship

$$\frac{2v_1}{(\sin\theta)t} = 0.05g$$

Substituting for v_1 using Equation 1-8 and solving for *R* gives the condition for the desired accuracy

$$R = 0.00031\, g \sin\theta\, t^2$$

For a *t* value of 4.5 seconds, and $\sin\theta$ equal to 0.1, we obtain *R* equal to 0.0063 meters, or about one quarter-inch. Students should plug in their own numbers to convince themselves that their results are affected by about this amount. They should compare this error to the effect of the uncertainty in the length measurement noted above in the discussion of significant figures.

Diluting Gravity 2

Experimental Procedure

In the second procedure the student employs a technology called the sonic ranging sensor that has been in use in physics classrooms for two decades. The sonic ranging sensor determines the distance to an object by measuring the time it takes for an ultrasonic pulse to travel from the sensor to the target and back again. Good results are obtained for flat objects that are perpendicular to the direction of propagation of the ultrasonic pulses, as long as these objects are at least 5 cm by 5 cm. This usually means that a small cardboard flag must be mounted on the air track glider to provide a flat target for the sensor beam. The sensors often have a dead spot, or region where measurements are inaccurate, which extends from the sensor out to a distance of 20 cm.

With these restrictions in mind, using the sensor is straightforward. It is best to mount the sensor at the raised end of the track, so that the glider moves further away with the passage of time. The glider is released, and then the sensor is triggered so that the computer can begin acquiring data. The data appear on the computer monitor, and may be fit to a quadratic equation using the curve-fitting program available with the data acquisition software. The coefficient of the linear term is equal to the velocity of the glider at the beginning of the data acquisition interval. The coefficient of the quadratic term is equal to $(g\sin\theta)/2$. Data are easy to acquire and analyze using this apparatus, and several trials may be run with gliders of different mass to show that the acceleration is independent of the glider's mass.

In addition to curve fitting, data acquisition programs often allow students to differentiate the data, so that the differential of the displacement with respect to time may be displayed. Physics C students should use this feature first to obtain a velocity versus time graph, then acceleration versus time graph. When they obtain these graphs, they see that the velocity versus time graph is not a smooth linear graph with a positive slope, and the acceleration versus time graph is not a constant with respect to time. This leads to a discussion of how the computer processes the data when it differentiates numerically, and the apparent discrepancy that results when compared to the expected graphs of velocity and acceleration versus time. That discussion is useful in showing the students the power and the limitations of their data processing program. They should see that curve fitting, since it is a best fit to all of the data, is a better method of obtaining the glider's acceleration than two successive numerical differentiations. These numerical differentiations can introduce artifacts into the data that can obscure the underlying phenomenon of motion with constant acceleration.

If students begin the lab with the working hypothesis that the glider is undergoing constant acceleration, one test of that hypothesis is to fit the data to a higher order polynomial. Students can use their data processing software to fit their data to a third-degree or higher order polynomial. The coefficients of the terms that are of cubic order or higher should be very small compared to the linear and quadratic terms. If the quadratic term is the highest significant term, this indicates that the object is undergoing constant nonzero acceleration.