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Sample Activity: *Accumulating Distance When Speed Is Constant*

from the Pre-AP workshop

Pre-AP®: Strategies in Mathematics – Accumulation

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Pre-AP: Strategies in Mathematics – Accumulation
Sample Activity

Accumulating Distance When Speed Is Constant

An important instructional goal in AP Calculus, given in the College Board's course description for *Calculus: Calculus AB, Calculus BC*, is:

> Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.

Our first step in this workshop is to identify mathematics content that middle school students need to learn for understanding the process of accumulation as it is applied to distance traveled for an object in motion. Our setting will be constant speed and our approach will be to explore how distance accumulates when speed is constant using multiple representations (i.e., words, tables, graphs, and symbols) as our tools for this exploration.

A guided exploration, a classroom-ready lesson appropriate for middle school students, will guide students in exploring constant speed using multiple representations. This guided exploration also presents a classroom pedagogical model for embedding diagnostic assessment within the practice of instruction. As you investigate this guided exploration, here are some questions to keep in mind and to discuss as a group:

1. At what grade level can this guided exploration be used?

2. a. Building an Answer Key:

   How would you respond to the questions in this guided exploration?

   b. Building a Diagnostic Assessment Key:

   What variety of responses would the questions in this guided exploration elicit from students? Attach, if possible, a grade level to each response.

   What would these various responses indicate about a student's mathematics understanding or lack of understanding?

   What follow-up questions or what kind of follow-up instruction would you use to increase student understanding based on a given response?

3. How do the mathematical ideas presented in this guided exploration anticipate "the meaning of the definite integral ... as the net accumulation of a rate of change"?

A Multiple Representational Look at Distance and Constant Speed

Part 1: When Speed Is Constant

Let's investigate the various ways to use mathematics to describe a trip by car. Speed is usually measured on a car's speedometer in miles per hour. In our investigation, distance will be measured in miles and time will be measured in hours.

We begin by comparing two different travel situations.

Situation 1: A car travels at a constant speed of 30 mph.

Situation 2: A car travels at a constant speed of 50 mph.

1. Indicate the distance traveled for each of the corresponding times by completing the tables below for each situation.

Situation 1			**Situation 2**	
Time (in hours)	Speed (mph)		Time (in hours)	Speed (mph)
0			0	
1			1	
2			2	
3			3	
4			4	
5			5	

2. Describe, using words or symbols, a pattern you see in each of the tables given in question 1. Describe your pattern in the corresponding spaces below.

Situation 1 Rule *(in words or symbols)*		**Situation 2 Rule** *(in words or symbols)*

3. Use the tables in question 1 or your description in question 2 to answer the following questions about the corresponding travel situation.

   a. How far has the car in Situation I gone after 3 hours?

   b. How far has the car in Situation 2 gone after 4 hours?

c. How long will it take the car in Situation I to travel 120 miles?

d. How long will it take the car in Situation 2 to travel 100 miles?

4. When planning for an extended trip by car, it would be helpful to have a method for directly computing the distance traveled without having to work out a table of values for intermediate times.

a. **Situation 1**

Write a rule, in the form of an equation $D =$ ________________ for the distance traveled by the car in Situation 1, with D denoting the distance traveled in miles and t denoting time in hours.

Use your rule to find the distance traveled after $t = 16$ hours by the car.

Use your rule to find the distance traveled after $t = 2.5$ hours by the car.

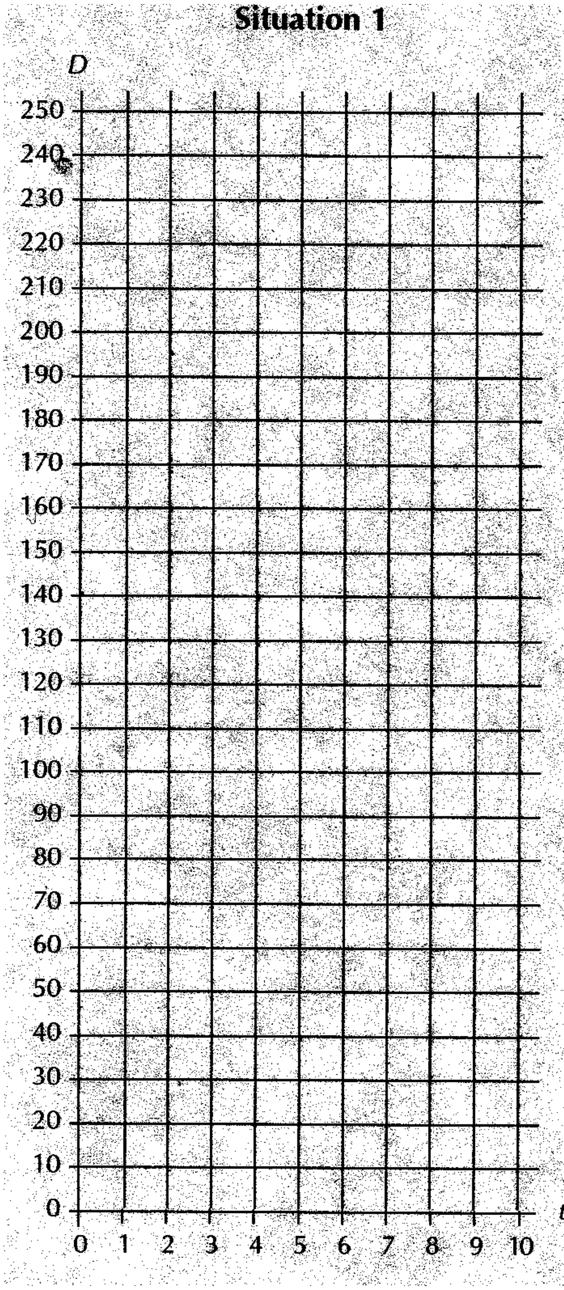
b. **Situation 2**

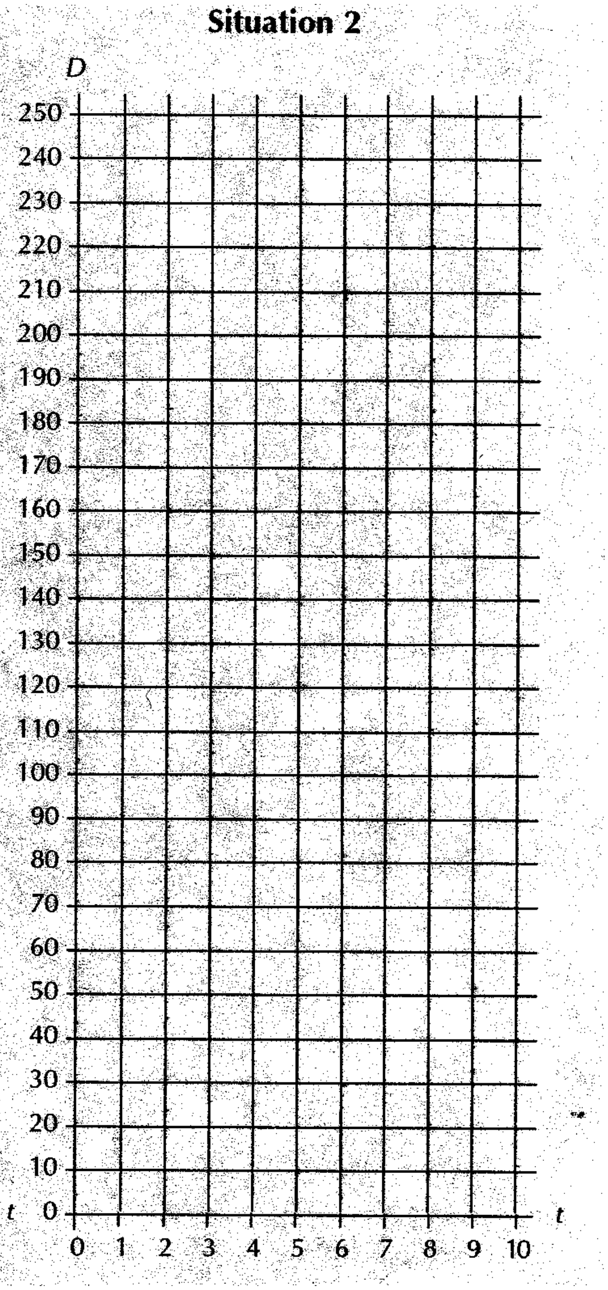
Write a rule, in the form of an equation $D =$ ______________ for the distance traveled by the car in Situation 2, with D denoting the distance traveled in miles and t denoting time in hours.

Use your rule to find the distance traveled after $t = 16$ hours by the car.

Use your rule to find the distance traveled after $t = 12.25$ hours by the car.

5. Expand the tables in question one to 10 hours and plot the values in each table as points on a coordinate system. Use one coordinate system for Situation 1 values and the other coordinate system for Situation 2 values. The horizontal axis denotes time in hours and the vertical axis denotes distance traveled in miles.





6. a. Describe ways in which the two plots in question 5 are **alike** and are **different**.

b. How is speed represented on these graphs?

Teacher Notes

In this section, we present a two-part guided exploration, appropriate for use at the middle school level that utilizes multiple representations to investigate the relationship of distance traveled to constant speed. This guided exploration assumes that students are at an early stage of development in their understanding and use of variables and algebraic expressions. The setting in this guided exploration emphasizes the algebraic relationship of distance traveled, D, in miles, in terms of elapsed time, t, in hours, given by the direct variation relationship $D = rt$. In the relationship $D = rt$, r is a constant denoting speed in miles per hour.

Before Using this Guided Exploration. Your students, without a doubt, have had many opportunities to travel by a variety of vehicles, e.g., cars, buses, trucks, trains, airplanes, bikes, scooters, skateboards, etc. In fact, a good question to use to introduce students to this guided exploration is to ask students by what types of vehicles they have traveled. Most students will think of automobiles, buses, and trucks. You may have to bring up bikes and other similar vehicles with a follow-up question.

Even though all of your students may have traveled in a car, do not assume, however, that all of them have a clear understanding of what "miles per hour" means. Many people, when traveling by car, often do not give much attention to the car's speed or rate of change. Their notion of a trip is simply getting from point A to point B without being attentive to an in-between trip detail like speed. Thus, it is important to open this lesson with some classroom demonstrations that show students that speed is rate of change.

For example, if your classroom floor or school halfway has square tiles, use these tiles to demonstrate the meaning of the rates of change of 3 tiles per second and 5 tiles per second. Make sure that students realize that speed is a rate of change in these demonstrations.

The rates of change in the setting of this guided exploration are 30 miles per hour and 50 miles per hour. Make sure that you relate the rate of change of 3 tiles per second to 30 miles per hour by asking students to discuss what is the same and what is different about these two rates of change. Giving class time to an opening demonstration and a follow-up discussion of rate of change/speed will provide important background understanding to this guided exploration that will benefit all students.

Part 1: When Speed Is Constant

1. The completed tables for each situation are as follows:

Situation 1			Situation 2	
Time (in hours)	Speed (mph)		Time (in hours)	Speed (mph)
0	0		0	0
1	30		1	50
2	60		2	100
3	90		3	150
4	120		4	200
5	150		5	250

This question is a diagnostic that provides a teacher with a simple and quick assessment that indicates how well students understand the rate of change term miles per hour and the setting of this guided exploration. If student entries differ from the entries given in the tables above, your first step should NOT be to correct these entries. Instead, your first step should be to ask students to explain how they came up with their entries. The explanations they give will help you understand what their difficulties are. When you understand the nature of their difficulties, you are better able to provide students with the necessary instruction to raise their level of understanding.

2. The types of answers that students will give to this question will vary with their grade level and knowledge. Here is a partial listing of some types of answers students will give for the pattern. (We illustrate their answers for the pattern for Situation 1. Patterns given for Situation 2 will be a natural modification of their Situation 1 patterns.) This list of answers is arranged in an order indicating the algebraic knowledge level of the student, ranging from a high algebraic level of understanding to a numerical level of understanding.

High Algebraic Level of Understanding

- $D = 30t$ or $t \bullet 30 = D$
- The distance in Situation 1 is equal to 30 mph multiplied by the time.
- Every hour the car travels 30 more miles.
- $30t$ *or* $t \bullet 30$
- Multiply the time by 30.
- Add by 30; counting by 30.
- Counting by 3 and placing a 0 at the end.
- $0 \times 30 = 0;\quad 1 \times 30 = 30;\quad 2 \times 30 = 60;$
  $3 \times 30 = 90;\quad 4 \times 30 = 120;\quad 5 \times 30 = 150$

Numerical Level of Understanding

- The type of answer that a student gives to this question provides a teacher with valuable diagnostic feedback about the student's level of understanding of patterns and pattern recognition, the concept of variable, and algebraic relationships of variables. An effective classroom teacher will use this diagnostic feedback to challenge and encourage students to a higher level of individual understanding with appropriate follow-up questions.

3. The intent of this question is to provide a set of simple questions that encourages students to reflect on the setting and to use the table of data. Teachers who use this guided exploration should emphasize the use of appropriate units by students when they give answers to these questions.

   a. The car in Situation 1 has gone 90 miles after 3 hours.

   b. The car in Situation 2 has gone 200 miles after 4 hours.

   c. It will take the car in the first situation 4 hours to travel 120 miles.

   d. It will take the car in the second situation 2 hours to travel 100 miles.

4. Students at an early stage in their development and understanding of the concept of variable and relationships between variables win struggle with this question. Students enter middle school grades with a very computational view of numbers. Questions asking them to express a pattern of numbers in a table are an excellent way to begin the process of developing their understanding of variables and relationships between varying quantities.

   Students with more developed algebra knowledge will be the ones who are able to give an answer in symbols in question 2. However, some of these students will have answered $30t$ instead of $D = 30t$. In this case, you have an opportunity to discuss time as the input quantity and distance as the output quantity. This begins to lay important groundwork for building their concept of function and the role of independent and dependent variables in the concept of function.

   It may be necessary to help some students craft an answer for the algebraic rule for Situation 1. This can usually be done by working with them on the table and getting them to see that there is a pattern, not just vertically on the distance column, but horizontally involving time as well. Once you have given sufficient help, challenge them to come up with the algebraic rule for Situation 2 on their own.

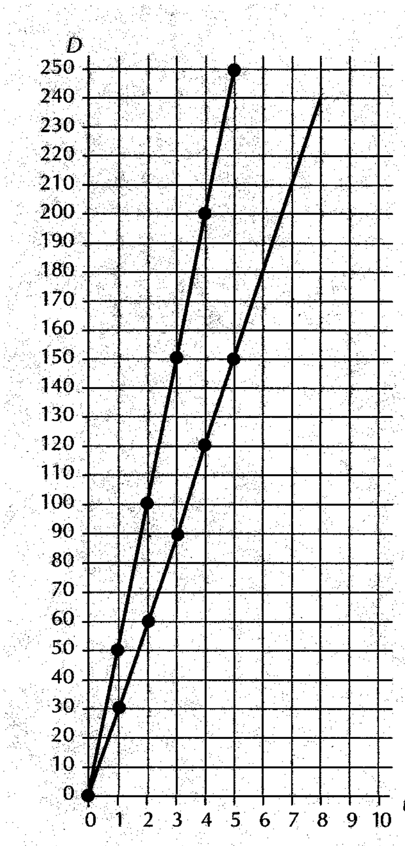
   Generally speaking, when middle school students do respond with an algebraic rule for these situations on their own, their rule will look as follows:

   $$t \times 30 = D \text{ or } t \times 30$$

   In algebra, we usually write this rule as $D = 30t$. The reason we do so is that we are interested in finding the output variable and, consequently, we desire a rule for the output variable in terms of the input variable. Middle school students tend to be more literal about the pattern they see in the table. They literally see "time times 30 equals distance" and that is why they write $t \times 30 = D$ instead of $D = 30t$. Their way of writing this relationship may look awkward to us - but it is, nevertheless, correct.

5. Students are asked to plot the values from the tables in question 1 as points on a coordinate system. A plot for the values in the tables for Situation 1 and Situation 2 on a single coordinate system is given below. This plot can be used as a "blackline" master to make an overhead transparency. This transparency can be an excellent resource for promoting discussion on the parts given in question 6.

   There are some things to watch for and discuss as students carry out the plot of values, from the tables in question 1. Some students will plot these values as discrete points, i.e., they will not connect these points. Others will plot these points, then take a straight edge and draw a line that connects these points. If these two types of plots occur in class, you have an excellent opportunity to discuss with the whole class these two answers and the issue of domain and range for the algebraic expressions $D = 30t$ and $D = 50t$ as it relates to describing the setting of this guided exploration. In this setting of distance and time, the domain for t is all values $t \geq 0$ and the range for D is all values $D \geq 0$. For most middle school students the words *domain* and *range* will be new and should be treated as new additions to their mathematical vocabulary. Domain, they should be told, is attached to the values of the input variable that are appropriate for the setting. The input variable of an algebraic expression is also called the independent variable. Range is attached to the values of the output variable that correspond to the values of the domain for the setting. The output variable of an algebraic expression is also called the dependent variable.



   If you are using this guided exploration with early middle school students, you will notice that some will be in need of remediation help with their skill in plotting. Typical errors that you may encounter include: (1) students who count spaces instead of the grid lines; (2) students who count grid lines but start their count with 1 on the vertical D-axis instead of 0; and (3) students who interchange the coordinates, i.e., count time along the vertical and distance along the horizontal. These mistakes, when they occur, do not take long to correct.

6. a. Ways in which these graphs are alike are that they are both line graphs and both pass through the origin. Some students may also offer as a similarity that both graphs rise.

   b. Will students recognize that the slope of these graphs represents speed? Many middle school students may not yet have had “slope" of a line as a formal topic in their mathematics class. For such students, slope is not yet a part of their mathematics vocabulary. They may use "steep" or "steepness" to describe how speed is represented in these graphs. If they do, this is an important step in their understanding of graphs.

   c. A simple way to articulate an answer to this question is “the steeper the graph the faster the speed" or "steeper means faster." This is a “big idea" of algebra. There are other ways that students can articulate how the speed in one situation is faster than in the other. For example, they can indicate that for every 1-hour movement to the right the graph goes up 50 miles in one and 30 miles in the other. Also, some may take a specific time value and indicate that on the one graph the distance is greater (higher) than the distance on the other.