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Sample Activity: *The Scene on the Jigsaw Puzzle Box Top*

from the Pre-AP workshop

Pre-AP®: Strategies in Mathematics – Rate

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Pre-AP: Strategies in Mathematics – Rate
Sample Activity

The Scene on the Jigsaw Puzzle Box Top

In a standard high school mathematics curriculum, the discussion of rates builds slowly. In the middle grades, students often investigate contexts in which the rate of change is constant. By extending observed patterns, middle school students answer questions about these contexts. In an algebra course, students learn to create linear functions to model contexts in which the rate of change is constant. As students move further along the mathematics curriculum, they encounter nonconstant rates of change such as those found in quadratic or exponential functions. Teachers use such contexts to help these more advanced students distinguish between constant and nonconstant rates of change. Seldom, however, are students asked to *find* nonconstant rates of change.

Where is the mathematical strand that focuses on the concept of rate of change heading? For example, what additional knowledge of rates of change would a student of calculus be expected to have and to use? In the language of our jigsaw puzzle analogy, what does the fully assembled puzzle look like?

The activity that follows has two parts. The first could be used in many high school classrooms. You will be asked to work on this with your group. The second is not intended for use in a high school mathematics classroom (unless the students are in an AP® Calculus class). Instead, it is intended to help all participants, no matter what their teaching assignments, understand how one might respond if asked to determine a rate of change that is not constant. You will work on this part with the presenter.

Part One: Getting Your Feet Wet

Consider the following table of data that shows the temperature of water in a pond at 3 p.m. as recorded every 3 days over a 15-day period. The first reading of 20°C was taken on May 31.

t (days)	$W(t)$ (°C)
0	20
3	22
6	24
9	26
12	28
15	30

1. Does the relationship between t and $W(t)$ appear to be linear? Explain why you answered as you did.

2. If this trend were to continue, on what date would the water temperature reach 50°C?

3. The data in the table shows that the water temperature has increased every three days since the May 31 reading. Assume that the temperature of the pond water has increased at a constant rate for this 15-day interval. Write an equation that describes the relationship between t and $W(t)$ over this 15-day interval.

4. What does the fact that $W(12) = 28$ tell us about this context? Include units in your answer.

5. How fast is the temperature of the pond water rising when $t = 12$? Be sure to include units in your answer.

6. How fast is the temperature of the pond water rising when $t = 14$? Explain how you arrived at your answer.

Teacher Notes

Rationale

Most of us who attempt to work a thousand-piece jigsaw puzzle have a strategy before we even begin. We put all of the pieces out on a table, we turn the pieces "right side up," and we place the box top in clear view. We want and need to know where we are heading.

Planning an effective curriculum involves a similar strategy. We must know the prerequisite skills, procedures, and concepts students will be expected to know and apply (the puzzle pieces) to meet both state and district requirements, and also to be effective citizens. As we help our students navigate the mathematics curriculum, we must take the individual topics on our list and begin to piece them together. We should always know not only where our students "have been" (i.e., what portions of the puzzle have already taken shape), but also where they "are headed" (i.e., what portions of the puzzle will need to be assembled in future courses).

For many of our students, a calculus course (either a high school AP course or one they take in college) will be in the future. In particular, early in the mathematics sequence it is not always clear which of our students will someday be enrolled in a calculus course, but we want them to be prepared so that they CAN enroll and be successful. Thus, it is in their best interests if we expect that ALL of them may choose to travel in that direction. Even more importantly, we want ALL of our students to be comfortable with the language and symbols of mathematics, to be confident about their skills, and to be able to apply their mathematical knowledge to contexts that a citizen is likely to encounter. The level of understanding expected of an AP Calculus student is a wonderful goal for us to have for all of our students.

Consider the following table of data that shows the temperature of water in a pond at 3 p.m. as recorded every 3 days over a 15-day period. The first reading of 20°C was taken on May 31.

t (days)	*W(t)* (°C)
0	20
3	22
6	24
9	26
12	28
15	30

1. Does the relationship between *t* and *W(t)* appear to be linear? Explain why you answered as you did.

A function is linear if and only if it has a constant rate of change. That is, each increment of the input generates a constant increment (either a constant increase or a constant decrease) in the output. Thus, the relationship between *t* and *W(t)* appears to be linear because every change of 3 in *t* generates a change of 2 in *W(t).*

Another way to say this is to mote that a linear function has a constant slope. Although we normally describe slope using the notation $\frac{\Delta y}{\Delta x}$, or $\frac{y_2-y_1}{x_2-x_1}$, the appropriate symbolic representation of the slope is $\frac{\Delta W}{\Delta t}$. For all possible ordered pairs, $(t, W(t))$ in our table, $\frac{\Delta W}{\Delta t} = \frac{2}{3}$.

Since this result arises from a situation involving days and water temperature, it shows greater understanding to write $\frac{\Delta W}{\Delta t} = \frac{2}{3}$ °C/day. This result is often interpreted as a change of 2°C every 3 days since May 31, but can also be interpreted as a chance of $\frac{2}{3}$ °C every day since May 31. Both interpretations are correct due to proportionality. It should be noted that the letter actually provides a unit rate of change which will be beneficial for students to recognize as they move through the curriculum toward the point at which they are introduced to the concept of instantaneous rate of change for nonlinear functions.

2. If this trend were to continue, on what date would the water temperature reach 50°C?

Some students, confident with symbolic representation, may anticipate question 3 and write that $W(t) - 28 = \frac{2}{3}(t-12)$ and that $W = 50$. The student who uses this (or an equivalent) equation would get:

$$50 - 28 = \frac{2}{3}(t-12)$$
$$\frac{3}{2}(22) = t - 12$$
$$33 + 12 = t$$
$$t = 45$$

Since t is the number of days after May 31, and since June has 30 days, the water temperature of the pond would reach 50°C forty-five days after May 31 - that is, on July 15.

Although students should be encouraged to show their work whenever possible, the need to "show work" sometimes precludes efficient approaches-based on core concepts. Having been asked question 2 before question 3, an insightful student may respond with the answer of $t = 45$ without believing that there is any work to be shown. Perhaps that particular student understands that proportionality is a component of the slope concept. If the student knows that $\frac{\Delta W}{\Delta t} = \frac{2}{3}$ is independent of the ordered pairs used, the student may recognize that when a 10°C change occurs over a 15-day period, a 30°C change must occur over a 45-day interval. Although a student using this reasoning could show the solution to the proportion $\frac{2}{3} = \frac{30}{t}$, he or she may not do so. A teacher will only know that the student has understood proportionality and applied that knowledge in question 2 ff the teacher does some probing. The resulting conversation would not only enlighten the teacher as to the student's level of understanding, but would also advance other students' understanding as well. For this reason, it is a good idea to ask that students either show their work or explain their reasoning.

This question will also allow the teacher to remind students to be sure they answer the question. Thus, a student who gets the answer $t = 45$ after showing his or her solution or explaining his or her reasoning is to be congratulated on the correct analysis of the situation and on finding the correct numerical value of t. Without the date of July 15, however, the question has not been answered. Students who write a complete sentence, using correct units, are to be treasured.

3. The data in the table shows that the water temperature has increased every three days since the May 31 reading. Assume that the temperature of the pond water has increased at a constant rate for this 15-day interval. Write an equation that describes the relationship between t and $W(t)$ over this 15-day interval.

In question 1, we acknowledged the fact that the data appeared to reflect a constant rate of change. Perhaps a more frequent recording of the pond water temperature would prove this assumption to be flawed. For example, maybe the water temperature dropped after the sun went down; in fact, a likely occurrence. In this question, we are asked to assume that the rate of change is constant.

In question 1 we found that the constant rate of change (i.e., the slope) was $\frac{2}{3}$. The first data entry of (0, 20) gives us the vertical axis intercept. Therefore, using the Slope-Intercept format for our linear function, we get $W(t) = \frac{2}{3}t + 20$. If we choose to use the Point-Slope format, we can use any one of the data values for our known point. Thus, one possible equation would be $W(t) - 28 = \frac{2}{3}(t - 12)$. If this activity were done in the classroom, this would be a wonderful time to be sure that students understand not only their options, but also that the various formats are equivalent. For example, have them show that

$W(t) - 28 = \frac{2}{3}(t - 12) \quad \Rightarrow$

$W(t) - 28 = \frac{2}{3}t - 8 \quad \Rightarrow$

$W(t) = \frac{2}{3}t + 20$

Most students are introduced to the Slope-Intercept format first, and. cling to it as they move through the curriculum. They will discover, however, that the Point-Slope format is easier to use in most applied problems. In fact, when written as $y = y_k + m(x - x_k)$, or $y = c_0 + c_1(x - a)$, the linear relationship is written as a first-degree Taylor polynomial. For this reason, some mathematics educators are encouraging algebra teachers to introduce the Point-Slope format as early as possible.

4. What does the fact that $W(12) = 28$ tell us about this context? Include units in your answer.

$W(12) = 28$ tells us that on June 12 (12 days after May 31) the temperature of the pond water is 28°C.

In order to answer this question, students must have been introduced to functional notation. For students who have not yet been introduced to functional notation, the question could be rephrased as the following: "What does the fact that $W= 32$ tell us about this context? Include units in your answer." However, when the question is posed this way it encourages a less complete answer because the student is not prompted to address $t = 12$. For example, a student might respond that "the water temperature is 28°C." In fact, the linearity of this function tells us that "the water temperature is 28°C (only) when t = 12."

5. How fast is the temperature of the pond water rising when $t = 12$? Be sure to include units in your answer.

The water temperature is rising at the rate of $\frac{2}{3}$ of a degree Celsius per day $\frac{2}{3}$ °C/day when $t = 12$ (on June 12 at 3 p.m.).

To answer this question correctly, students must recognize that the word fast relates to how W is changing with respect to t. The word rising suggests that this change is positive. If the question were to be reworded as "How fast is the temperature of the pond water changing when $t = 12$?" the best response would be that the temperature is increasing at the rate of $\frac{2}{3}$ °C/day on June 12 at 3 p.m.

6. How fast is the temperature of the pond water rising when $t = 14$? Explain how you arrived at your answer.

Students who have answered question 5 correctly must now only show that they truly understand what it means for the rate of change to be constant. Even though $t = 14$ did not appear in the table of data, the fact that the rate of change has been declared to be constant assures us that the answer to question 6 is the same as the answer to question 5. That is, the water temperature is rising at the rate of $\frac{2}{3}$ °C/day when $t = 14$ (on June 14 at 3 p.m.).

If a student were to respond that there is not enough information provided to answer this question, the teacher can use this question as an opportunity to help that student better understand both the phrase "constant rate of change" and how the phrase relates to linear functions.