AP® Calculus AB (Operational)
2004 Sample Student Responses

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Work for problem 3(a)

$$a(t) = v'(t) = \frac{-e^t}{1+e^{2t}}$$

$$\therefore a(2) = \frac{-e^2}{1+e^4} = \boxed{-.133}$$

Work for problem 3(b)

$$v(2) = 1 - \tan^{-1}(e^2) = -.436$$

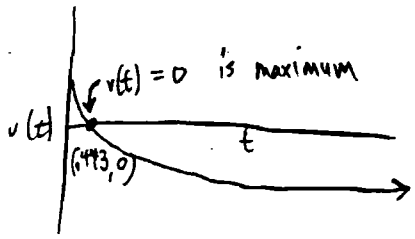
speed is increasing because
at $t = 2$, $v(t) < 0$ and $a(t) < 0$

Continue problem 3 on page 9.

Work for problem 3(c)

$v(t) = 0 = 1 - \tan^{-1}(e^t)$

$\therefore t = .443$ is a maximum
because $V(t) = y'(t) > 0$ on $0 \le t < .443$
and $v(t) = y'(t) < 0$ on $.443 < t < \infty$



Work for problem 3(d)

$y(2) = -1 + \int_0^2 v(t) = -1 + \int_0^2 1 - \tan^{-1}(e^t) = -1.361$

$v(2) = 1 - \tan^{-1}(e^2) = -.436$

$\therefore$ particle is moving away from origin
because at $t = 2$ both $y(t)$ and $v(t)$ are less than zero

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Work for problem 3(a)

$v(t) = 1 - \tan^{-1}(e^t)$

$a(t) = v'(t) = \dfrac{-1}{(e^t)^2 + 1} = \dfrac{1}{e^{2t} + 1}$

$a(2) = \dfrac{1}{e^{2(2)} + 1} = \dfrac{1}{e^4 + 1} \approx .018$

Work for problem 3(b)

$a(t) = \dfrac{1}{e^{2t} + 1}$

$a(2) = .018$

The speed of the particle is increasing at time $t = 2$ because $a(2) > 0$, which means that the particle is accelerating at $t = 2$.

Continue problem 3 on page 9.

C2

Work for problem 3(c)

(1) $v(t) = 1 - \tan^{-1}(e^t)$

$0 = 1 - \tan^{-1}(e^t)$

$1 = \tan^{-1}(e^t)$

$\tan(1) = e^t$

$\ln(\tan(1)) = t$

$.443 \approx t$

$v(.2) > 0$ (+) $\quad v(1) < 0$ (−)

0 .443

increasing decreasing

(NO other critical points)

The particle reaches its highest point at $t \approx .443$ because the velocity goes from positive to negative (graph stops increasing and starts decreasing) creating an absolute maximum.

Work for problem 3(d)

(a) $s(2) = s(0) + \int_0^2 v(t)\,dt$

$= (-1) + \int_0^2 (1 - \tan^{-1}(e^t))\,dt$

$\approx (-1) + (-.361)$

$\approx \boxed{-1.361 \text{ un}}$

The particle is moving away from the origin at $t = 2$ because $v(2) < 0$.

END OF PART A OF SECTION II (see sign chart 3(c))

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