AP® Calculus BC
Syllabus 4

Calculus pulls together many of the concepts students have studied in previous courses, and it also helps them to see the relevance of the material they were taught prior to calculus. I believe that AP® Calculus BC gives the students a strong foundation for the math and science courses they will take in college.

Course Planner [C2]

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and Series as delineated in the Calculus BC Topic Outline in the *AP Calculus Course Description*.

First Semester

Chapter 1: Limits and Their Properties (10 days—one test) [C2]

- Lab on limits
- An introduction to limits, including an intuitive understanding of the limit process
- Using graphs and tables of data to determine limits
- Properties of limits
- Algebraic techniques for evaluating limits
- Comparing relative magnitudes of functions and their rates of change
- Continuity and one-sided limits
- Geometric understanding of the graphs of continuous functions
- Intermediate Value Theorem
- Infinite limits
- Using limits to find the asymptotes of a function

Chapter 2: Differentiation (19 days—two tests) [C2]

- Zooming-in activity and local linearity
- Understanding of the derivative: graphically, numerically, and analytically
- Approximating rates of change from graphs and tables of data
- The derivative as: the limit of the average rate of change, an instantaneous rate of change, limit of the difference quotient, and the slope of a curve at a point
- The meaning of the derivative—translating verbal descriptions into equations and vice versa

- The relationship between differentiability and continuity
- Functions that have a vertical tangent at a point
- Functions that have a point on which there is no tangent
- Differentiation rules for basic functions, including power functions and trigonometric functions
- Rules of differentiation for sums, differences, products, and quotients
- The chain rule
- Implicit differentiation
- Related rates

Chapter 3: Applications of Differentiation (19 days—two tests) [C2]

- Extrema on an interval and the Extreme Value Theorem
- Rolle's Theorem and the Mean Value Theorem, and their geometric consequences
- Lab on the First Derivative Test
- Increasing and decreasing functions and the First Derivative Test
- Lab on concavity and points of inflection
- Concavity and its relationship to the first and second derivatives
- Second Derivative Test
- Limits at infinity
- A summary of curve sketching—using geometric and analytic information as well as calculus to predict the behavior of a function
- Relating the graphs of f, f', and f''
- Optimization including both relative and absolute extrema
- Tangent line to a curve and linear approximations
- Application problems including position, velocity, acceleration, and rectilinear motion

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Chapter 4: Integration (15 days—two tests) [C2]

- Antiderivatives and indefinite integration, including antiderivatives following directly from derivatives of basic functions
- Basic properties of the definite integral
- Area under a curve
- Meaning of the definite integral
- Definite integral as a limit of Riemann sums
- Riemann sums, including left, right, and midpoint sums
- Trapezoidal sums
- Use of Riemann sums and trapezoidal sums to approximate definite integrals of functions that are represented analytically, graphically, and by tables of data
- Discovery lesson on the First Fundamental Theorem of Calculus
- Use of the First Fundamental Theorem to evaluate definite integrals
- Use of substitution of variables to evaluate definite integrals
- Integration by substitution
- Discovery lesson on the Second Fundamental Theorem of Calculus
- The Second Fundamental Theorem of Calculus and functions defined by integrals
- The Mean Value Theorem for Integrals and the average value of a function

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Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions (16 days—two tests) [C2]

- The natural logarithmic function and differentiation
- The natural logarithmic function and integration
- Inverse functions
- Exponential functions: differentiation and integration
- Bases other than e and applications
- Solving separable differential equations
- Applications of differential equations in modeling, including exponential growth

- Use of slope fields to interpret a differential equation geometrically
- Drawing slope fields and solution curves for differential equations
- Euler's method as a numerical solution of a differential equation

First Semester Exam (two review days)

Second Semester

Chapter 5: Logarithmic, Exponential, and Other Transcendental Functions (4 days—one test) [C2]

- Inverse trig functions and differentiation
- Inverse trig functions and integration

Chapter 6: Applications of Integration (10 days—one test) [C2]

- The integral as an accumulator of rates of change
- Area of a region between two curves
- Volume of a solid with known cross sections
- Volume of solids of revolution
- Arc length
- Applications of integration in physical, biological, and economic contexts
- Applications of integration in problems involving a particle moving along a line, including the use of the definite integral with an initial condition and using the definite integral to find the distance traveled by a particle along a line

Chapter 7: Integration Techniques, L'Hôpital's Rule, and Improper Integrals (17 days—two tests) [C2]

- Review of basic integration rules
- Integration by parts
- Trigonometric integrals
- Integration by partial fractions
- Solving logistic differential equations and using them in modeling
- Discovery lab on L'Hôpital's Rule

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- L'Hôpital's Rule and its use in determining limits
- Discovery activity on improper integrals
- Improper integrals and their convergence and divergence, including the use of L'Hôpital's Rule

Chapter 8: Infinite Series (17 days—two tests) [C2]

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- Lab on Sequences
- Convergence and divergence of sequences
- Definition of a series as a sequence of partial sums
- Convergence of a series defined in terms of the limit of the sequence of partial sums of a series
- Introduction to convergence and divergence of a series by using technology on two examples to gain an intuitive understanding of the meaning of convergence
- Geometric series and applications
- The *n*th-Term Test for Divergence
- The Integral Test and its relationship to improper integrals and areas of rectangles
- Use of the Integral Test to introduce the test for *p*-series
- Comparisons of series
- Alternating series and the Alternating Series Remainder
- The Ratio and Root Tests
- Taylor polynomials and approximations: introduction using the graphing calculator
- Power series and radius and interval of convergence
- Taylor and Maclaurin series for a given function
- Maclaurin series for $\sin x$, $\cos x$, e^x, and $\frac{1}{1-x}$
- Manipulation of series, including substitution, addition of series, multiplication of series by a constant and/or a variable, differentiation of series, integration of series, and forming a new series from a known series
- Taylor's Theorem with the Lagrange Form of the Remainder (Lagrange Error Bound)

Chapter 10: Plane Curves, Parametric Equations, and Polar Curves (12 days—one test) [C2]

- Plane curves and parametric equations
- Parametric equations and calculus
- Parametric equations and vectors: motion along a curve, position, velocity, acceleration, speed, distance traveled
- Analysis of curves given in parametric and vector form
- Polar coordinates and polar graphs
- Analysis of curves given in polar form
- Area of a region bounded by polar curves

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After the AP Exam:

Chapter 16: Differential Equations (6 days—one test)

- Definitions and basic concepts of differential equations
- First-order linear differential equations
- Second-order homogeneous linear equations

Chapter 5: Hyperbolic Functions (4 days—one test)

- Hyperbolic functions and applications

Second Semester Exam (two review days)

Teaching Strategies

Learning by Discovery

I introduce each unit with a discovery lesson. I think that exploration and discovery are great ways for students to learn, because these methods help students have more ownership in the material being covered than they would using a traditional lecture approach. The discovery lessons are done in groups of two or three students.

Graphing Calculator

Many of the discovery lessons rely heavily on the use of the graphing calculator. The calculator helps students develop a visual understanding of the material that they would not otherwise have. My students use the TI-89 graphing calculator almost every day in class for explorations and in assignments to analyze functions and justify solutions. For example, the students use the calculator to approximate the values of derivatives and definite integrals obtained through analytical means in order to verify that the answers are reasonable. [C5]

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

However, many homework problems and about half of the problems on quizzes and tests are done without the use of the graphing calculator. Since the AP Exam is half calculator and half noncalculator, I feel that it is very important for students to have practice working problems both ways. We spend time in class discussions talking about the types of questions that they must know how to work *with* their calculators and the types of questions that they must know how to work *without* their calculators. We also discuss the techniques needed to use the calculator most efficiently (storing functions in the $y =$ screen, storing values that will be used later in the problem, etc.).

Rule of Four

I give my students many opportunities to work problems presented in a variety of ways: graphical, numerical, analytical, and verbal. Most of the problems in my primary textbook are written with an analytical representation, so I frequently supplement these problems with ones that utilize a graph or tabular data. I also often ask students for verbal explanations to give them the opportunity to communicate their reasoning in words. **[C3]**

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

Justification of Answers

I ask my students to justify their answers on homework, quizzes, and tests, and I prefer that they write the justifications in sentences. We talk a lot about the amount of work they need to show and the correct way to justify their work on various types of problems. **[C4]**

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

(The "Commentary on the Instructions for the Free-Response Section of the AP Calculus Exams" on AP Central® is very helpful in showing examples of correct justifications.)

AP Review

I try to allot a minimum of three weeks before the AP Exam to devote to review. During this three-week period, students work on the sample questions in the *AP Calculus Course Description* and on multiple-choice and free-response questions from AP Released Exams. Some of these are assigned for homework, while others are given as a quiz or test.

Primary Textbook

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus with Analytic Geometry*. 5th ed. Boston: Houghton Mifflin, 1994.