AP® Calculus BC
Syllabus 2

Teaching Strategies

Classroom Dynamics

Because of my strong belief that students learn best by discovering new concepts for themselves, I attempt to promote an atmosphere of questioning, exploration, and excitement in the classroom. Rarely does a lesson proceed straight down a prepared path; we take frequent side trips. I encourage students to ask "what if" questions, for which I often do not have ready answers. The objective is to engage students in enjoyable activities that promote interest in mathematics. I try to get them to ask the questions. I rarely, if ever, tell students that some new concept or type of problem is easy. I'd rather they feel a sense of accomplishment from being able to tackle hard concepts and problems than feel frustration at being stumped by even the easy ones.

One consequence of calculus reform and of the accessibility of technology is that questions are becoming much more interesting and diverse. The more experience students have with solving interesting and difficult problems, the better, both for the AP® Examinations and in the long run.

Assessment

The issue of assessment in a technology-intensive classroom is one that teachers must resolve intelligently. My own approach is to allow the use of graphing calculators on nearly all unit tests. Before the AP Exam, I make sure the students are proficient at using technology to perform the four basic activities required on it: graphing a function in an arbitrary window, finding roots and points of intersection, finding numerical derivatives, and approximating definite integrals. Students are often directed to use the calculator to investigate concepts such as limits by using the trace and table operations to make conjectures about the answers. They are also frequently asked to use the calculator to approximate answers found algebraically to see if they are reasonable. **[C5]**

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions

Laboratory Activities

For each major content area, students are introduced to new topics through group work using discovery-learning activities.

Calculus Journal

Students are also required to keep a Calculus Journal. Questions are given in class to which students respond in their journals. For instance, one question this year was, "What is the most important concept we've learned in calculus so far? Justify your answer." Another was, "Explain, in your own words, what the first Fundamental Theorem of Calculus says." Students are encouraged to write frequently in their journals. **[C4]**

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

Major Themes

For each new major idea, I attempt to examine the concept graphically, numerically, and symbolically, and I illustrate connections among the three. I am also attentive to students' verbal expression of concepts, and make repeated and determined efforts to encourage them to be precise in their use of language. We use graphing calculators throughout the course. **[C3] [C4] [C5]**

AP Calculus BC Course Outline [C2]

Preliminary

Students who begin Calculus BC have already had experience using graphing calculators. Nonetheless, time is spent at the beginning of the course addressing issues of the limitations of technology, including round-off error, hidden behavior examples, and other issues.

Unit I: Functions (12 days)

Lab: Exploring function transformations $f(x + h)$, $f(x) + k$, $a*f(x)$, $f(b*x)$, $f(|x|)$, $|f(x)|$

- Multiple representations of functions
- Absolute value and interval notation
- Domain and range
- Categories of functions, including linear, polynomial, rational, power, exponential, logarithmic, and trigonometric
- Even and odd functions
- Function arithmetic and composition
- Inverse functions
- Parametric relations

Unit II. Limits (11 days)

Lab: Computing limits graphically and numerically

- Informal concept of limit
- Language of limits, including notation and one-sided limits
- Calculating limits using algebra
- Properties of limits
- Limits at infinity and asymptotes

C3—The course provides students with the opportunity to work with functions represented in a variety of ways—graphically, numerically, analytically, and verbally—and emphasizes the connections among these representations.

C4—The course teaches students how to communicate mathematics and explain solutions to problems both verbally and in written sentences.

C5—The course teaches students how to use graphing calculators to help solve problems, experiment, interpret results, and support conclusions.

C2—The course teaches all topics associated with Functions, Graphs, and Limits; Derivatives; Integrals; and Polynomial Approximations and Series as delineated in the Calculus BC Topic Outline in the *AP Calculus Course Description*.

- Estimating limits numerically and graphically
- Comparing growths of logarithmic, polynomial, and exponential functions
- Idea of continuity and the limit definition
- Types of discontinuities
- The Intermediate Value and Extreme Value Theorems
- Local and global behavior
- Rate of change concept
- Tangent lines, including using the tangent line to approximate a function
- Formal definitions of limit and continuity

Unit III. The Derivative (25 days)

Lab: The derivative and differentiability

- Linear functions and local linearity
- Slope–intercept, point slope, and Taylor forms of linear equations
- Difference quotient definition of derivative; computing the derivative at a point using the definition
- Estimating the derivative from tables and graphs
- Relationship between differentiability and continuity
- Symmetric difference quotient definition
- The derivative as a function; computing derivative functions from the definition
- Derivative as a rate of change
- Rules for computing derivatives; formulas for all relevant functions, including implicitly defined functions

Unit IV. Applications of Derivatives (17 days)

Lab: An Investigation into the Accuracy of the Tangent Line Approximation

- Finding extrema
- Increasing and decreasing behavior
- The Mean Value Theorem
- Critical values and local extrema
- The first and second derivative tests
- Concavity and points of inflection
- Comparing graphs of f, f', and f''
- Modeling and optimization
- Particle motion; position, velocity, and acceleration functions
- Linearization and the Taylor form of the equation of a line
- Newton's method
- Related rates problems

Review for Semester Exam (5 days)

Unit V. The Definite Integral (22 days)

"Car" Lab: Speedometer readings and distance traveled

Lab: Accumulation Functions (from College Board Professional Development Workshop) Materials Special Focus: The Fundamental Theorem

Lab: Riemann Sums (from Texas Instruments' *Calculus Activities*)

Lab: The Fundamental Theorem (from College Board Professional Development Workshop Materials Special Focus: The Fundamental Theorem

- Area under a curve and distance traveled
- Summation notation and partitions
- Riemann sum
- Definition of the definite integral as the limit of a Riemann sum
- Linearity properties of definite integrals

- Average value of a function
- Definition of antiderivative
- The idea of area function; discovering the fundamental theorem
- The First and Second Fundamental Theorems of Calculus and their uses
- The Mean Value Theorem for Integrals, and using the Fundamental Theorem to connect the two Mean Value Theorems
- Numerical integration techniques: left endpoint, right endpoint, midpoint, trapezoid, and Simpson's rules

Unit VI. Differential Equations and Mathematical Modeling (24 days)

Lab: Using Slope Fields (from Texas Instruments *Calculus Activities*)

- Initial value problems
- Translating verbal descriptions into differential equations
- Antiderivatives and slope fields
- Linearity properties of definite integrals
- Techniques of antidifferentiation: substitution and integration by parts
- Solving separable differential equations analytically
- The domain of the solution of a differential equation
- Exponential growth problems
- The logistic model and antiderivatives by partial fractions
- Solving initial value problems by Euler's method
- Solving initial value problems visually using slope fields
- Solving initial value problems using the Fundamental Theorem

Unit VII Applications of Definite Integrals (23 days)

- Integral of a rate of change gives net change
- Measuring area under and between functions; Cavalieri's principle

- Measuring volume of solids of known cross-sectional area and solids of revolution
- Applications to particle motion—net and total distance traveled
- Arc length of function graphs

Review for Semester Exam (5 Days)

Unit VIII. Parametric, Vector, and Polar Functions (17 days)

- Length of parametrically defined curves
- Vectors and vector-valued functions
- Calculus of vector functions
- Calculus of polar functions, including slope, length, and area

Unit IX. Sequences (12 days)

- Idea and notation for sequences; arithmetic, harmonic, alternating harmonic, and geometric sequences
- Definitions of convergence and divergence
- Bounded, monotonic, oscillating sequences
- Limit properties of sequences
- L'Hôpital's Rule and indeterminate forms
- Relative rates of growth of functions
- Improper integrals and the comparison test

Unit X. Series (24 days)

Lab: An Investigation into the Accuracy of Polynomial Approximations to Transcendental Functions

- Definition and notation of series; sequence of partial sums; telescoping, geometric, harmonic, alternating harmonic series
- Repeating decimals expressed as infinite geometric series; using substitution and antidifferentiating to calculate series for ln(1+x) and arctan(x) from geometric series
- Terms of series as areas of rectangles; relationship to the integral test

- Power series; interval and radius of convergence defined
- Taylor series
- Maclaurin series for e^x, sin x, cos x, and $\frac{1}{1 - x}$
- Functions defined by series
- Taylor polynomials
- Taylor's theorem with Lagrange form of the remainder
- Alternating series error bound
- Linearity properties of series
- Radius of convergence: *n*th term test; direct comparison test; absolute and conditional convergence; ratio test
- Interval of convergence and testing endpoints; integral test; *p*-series; limit comparison test; alternating series test

References and Materials

Major textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic*. Reading, Mass.: Addison-Wesley, 2007.