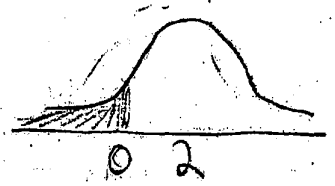
3B

3. The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

(a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?



ncdf(-1e99, 0, 2, 1.5)

P = .09121 that the value will be negative

(b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that at least one of these measurements will be negative?

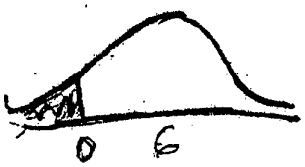
$_3C_1(.09121)^1(1-.09121)^2 = .22599$

$_3C_2(.09121)^2(1-.0912)^1 = .02269$

$_3C_3(.09121)^3(1-.0912)^3 = +7.588 \times 10^{-4}$

P = .2494 at least one will be negative

(c) What is the probability that the mean of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?



$\sigma = \sqrt{1.5^2+1.5^2+1.5^2}$

$\sigma = 2.598$

ncdf(-e99, 0, 6, 2.598)

P = .01046 that the mean will be negative

3 C

3. The depth from the surface of Earth to a refracting layer beneath the surface can be estimated using methods developed by seismologists. One method is based on the time required for vibrations to travel from a distant explosion to a receiving point. The depth measurement (M) is the sum of the true depth (D) and the random measurement error (E). That is, $M = D + E$. The measurement error (E) is assumed to be normally distributed with mean 0 feet and standard deviation 1.5 feet.

(a) If the true depth at a certain point is 2 feet, what is the probability that the depth measurement will be negative?

$M = D + E$

$M = 2 + E$

for M to be negative, $E < -2$

$$P(E < -2) = P\left(Z < \frac{-2-0}{1.5}\right) = P(Z < -1.33) = \boxed{0.0912}$$

(b) Suppose three independent depth measurements are taken at the point where the true depth is 2 feet. What is the probability that at least one of these measurements will be negative?

Let X be a binomial var, the number of measurements that are negative.

conditions for binomial: ① Fixed n # of observations – yes, 3
② P(success) same for all – yes
③ Independence of observations – yes
④ Only 2 outcomes – yes, + or –

$$P(X \geq 1) = 1 - P(X = 0)$$
$$= \boxed{0.249}$$

(c) What is the probability that the mean of the three independent depth measurements taken at the point where the true depth is 2 feet will be negative?

$$P\left(\frac{1}{3}(M_1 + M_2 + M_3) < 0\right) = P(M_1 + M_2 + M_3 < 0) = P(6 + E_1 + E_2 + E_3 < 0)$$
$$= P(E_1 + E_2 + E_3 < -6)$$

this has mean $0+0+0 = 0$

$$\text{S.d.} = \sqrt{1.5^2 + 1.5^2 + 1.5^2} = 2.598$$

this probability = $\boxed{0.010}$

GO ON TO THE NEXT PAGE.

AP® STATISTICS
2006 SCORING COMMENTARY

Question 3

Overview

The primary goals of this question were to assess a student's ability to: (1) recognize the random variable of interest, identify its probability distribution, and calculate a probability for a linear combination of a normal random variable and a constant; (2) use basic probability rules to find a different probability; and (3) use the sampling distribution of the sample mean to find a probability about the mean of three observations.

Sample: 3A
Score: 4

In part (a) the depth measurement, *M*, is clearly indicated as the variable of interest. The notation is clear and consistent and includes a sketch showing the normality of the distribution of the depth measurement, the *z*-score, and the shaded tail region. In part (b) the complement of the answer in part (a) is used to calculate the probability that at least one of the depth measurements will be negative. Parameters are identified, and the steps in the solution are clear. In part (c) a drawing is used to show the normality of the sampling distribution. The sketch includes the *z*-score and the shaded tail region. The work for calculating the mean and standard deviation of the sampling distribution is clear. There is a minor notation error, $P(X < 0)$, that was ignored in evaluating the overall essay. The correct notation should be $P(\overline{X} < 0)$. This essay earned a score of 4.

Sample: 3B
Score: 3

In part (a) the distribution of error, *E*, is used as the variable of interest. Since the measurement error is also normally distributed, full credit is given for this response. A drawing is used to indicate the normality of the distribution, the value of the mean, and the shaded tail region. Calculator syntax is given to compute the correct probability, but the standard deviation is not specified. In part (b) the binomial distribution is used to calculate the correct probability that at least one of three independent depth measurements will be negative. The correct calculation is shown without using the word binomial, but the values of *n* and *p* are clear from the work. In part (c) the distribution of the sum of three measurements is used to calculate the probability. The question asked the student to calculate the probability that the mean of the three independent depth measurements taken at the point where the true depth is 2 feet would be negative, but since the sum of the three measurements is also normally distributed, this approach to the solution was given full credit. The correct value for the mean and standard deviation for the sum is given. A drawing is used to indicate the distribution is normal, the value of the mean, and a shaded tail region. All parameters are defined in the calculator syntax. This essay earned a score of 3.

Sample: 3C
Score: 2

In part (a) the distribution of error, *E*, is used as the variable of interest. Since the measurement error is also normally distributed, full credit was given for this response. The essay uses correct notation, calculates the correct probability, and identifies the value of the mean and standard deviation in the substitution step of the *z*-score formula. In part (b) the binomial distribution is used to calculate the correct probability that at least one of three independent depth measurements will be negative. The essay correctly states the conditions for a binomial distribution and calculates the correct probability, but the student fails to give the parameter value *p*. In part (c)

Question 3 (continued)

students were asked to calculate the probability that the mean of the three independent depth measurements taken at the point where the true depth is 2 feet would be negative, but since the sum of the three measurements is also normally distributed, this student's approach to the solution was given full credit. The values for the mean and standard deviation are labeled and the correct probability is calculated, but the fact that the distribution of the sum of three independent depth measurements is normal is not shown. Although correct probability is given, it is not clear how the answer is calculated. This essay earned a score of 2.