

5. (10 points)

A cylinder with a movable frictionless piston contains an ideal gas that is initially in state 1 at 1×10^5 Pa, 373 K, and 0.25 m^3 . The gas is taken through a reversible thermodynamic cycle as shown in the PV diagram above.

(a) Calculate the temperature of the gas when it is in the following states.

i. State 2 $PV = nRT$, at both states, nR equal

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad T_2 = \frac{P_2 V_2 T_1}{P_1 V_1}, \quad P_2 = P_1$$

$$T_2 = \frac{(0.50 \text{ m}^3)(373 \text{ K})}{(0.25 \text{ m}^3)} = 746 \text{ K}$$

ii. State 3

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \quad V_1 = V_3 \quad T_3 = \frac{P_3 T_1}{P_1}$$

$$T_3 = \frac{(1.5 \times 10^5 \text{ Pa})(373 \text{ K})}{(1 \times 10^5 \text{ Pa})}$$

$$= 560 \text{ K}$$

(b) Calculate the net work done on the gas during the cycle.

$$\begin{aligned}
 1 \rightarrow 2 \quad W_{12} &= -P\Delta V \\
 &= 1.0 \times 10^5 \text{ Pa} (0.50 - 0.25) \text{ m}^3 \\
 &= -25000 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 2 \rightarrow 3 \quad W &= \text{area under } 2 \xrightarrow{\text{line}} 3 = \frac{P(S_1 + S_2)}{2} \\
 &= \frac{(0.50 - 0.25)(1.0 \times 10^5 + 1.5 \times 10^5)}{2}
 \end{aligned}$$

$$\begin{aligned}
 &= +31250 \text{ J} \\
 3 \rightarrow 1 \quad &\text{no work}
 \end{aligned}$$

$$W_{\text{net}} = 31250 - 25000 = 6250 \text{ J}$$

(c) Was heat added to or removed from the gas during the cycle?

Added Removed Neither added nor removed

Justify your answer.

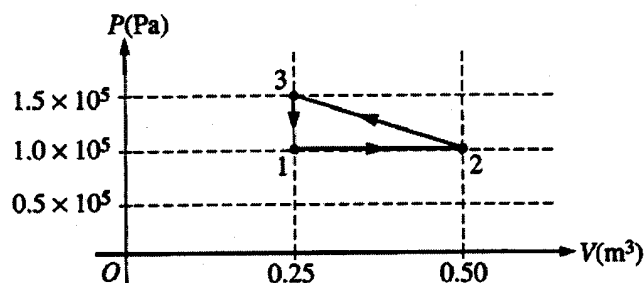
$\Delta U = 0$ since any cycle returns to its original state at the end.

$$\Delta U = \Delta Q + \Delta W$$

$$\Delta Q = -\Delta W$$

Since $W = +$, $Q = -$

\therefore heat removed



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(a) Calculate the temperature of the gas when it is in the following states.

i. State 2

$$PV = nRT$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{(1 \times 10^5 \text{ Pa})(0.25 \text{ m}^3)}{373 \text{ K}} = \frac{(1.0 \times 10^5 \text{ Pa})(0.5 \text{ m}^3)}{T_2}$$

$$T_2 = 746 \text{ K}$$

ii. State 3

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3}$$

$$\frac{P_1}{T_1} = \frac{P_3}{T_3}$$

$$\frac{(1 \times 10^5 \text{ Pa})}{373 \text{ K}} = \frac{(1.5 \times 10^5 \text{ Pa})}{T_3}$$

$$T_3 = 559.5 \text{ K}$$

(b) Calculate the net work done on the gas during the cycle.

$$W = -P\Delta V$$

$$W_{1 \rightarrow 2} = (1 \times 10^5)(0.25)$$

$$= 2.5 \times 10^4 \text{ J}$$

$$W_{2 \rightarrow 3} = 1.5 \times 10^5 (-0.25)$$

$$= -3.75 \times 10^4 \text{ J}$$

$$= -2.5 \times 10^4 \text{ J}$$

$$W_{3 \rightarrow 1} = (1.0 \times 10^5)(0) = 0$$

NET WORK

$$2.5 \times 10^4 \text{ J}$$

$$- 3.75 \times 10^4 \text{ J}$$

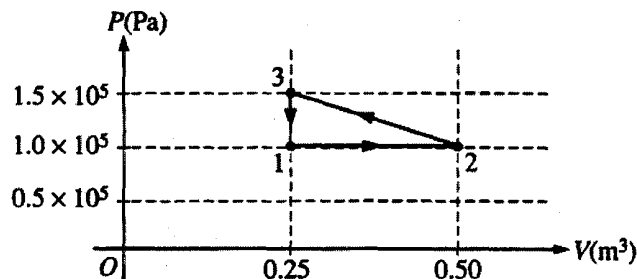
$$\boxed{-1.25 \times 10^4 \text{ J}}$$

(c) Was heat added to or removed from the gas during the cycle?

Added Removed Neither added nor removed

Justify your answer.

If work is negative, the system did work on its surroundings, and therefore gave up heat.



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(a) Calculate the temperature of the gas when it is in the following states.

i. State 2

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{(1.0 \times 10^5 \text{ Pa})(.25 \text{ m}^3)}{373 \text{ K}} = \frac{(1.0 \times 10^5 \text{ Pa})(.50 \text{ m}^3)}{x}$$

$$x \frac{(1.0 \times 10^5 \text{ Pa})(.25 \text{ m}^3)}{(1.0 \times 10^5 \text{ Pa})(.25)} = \frac{(373)(1.0 \times 10^5)(.50 \text{ m}^3)}{(1.0 \times 10^5)(.25)}$$

$$x = \frac{(373)(.50)}{.25} = 746 \text{ K}$$

ii. State 3

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{(1.0 \times 10^5)(.25)}{373} = \frac{(1.5 \times 10^5)(.25)}{x}$$

$$x (1.0 \times 10^5)(.25) = (373)(1.5 \times 10^5)(.25)$$

$$x = \frac{(373)(1.5 \times 10^5)(.25)}{(1.0 \times 10^5)(.25)} \quad x = 562.5 \text{ K}$$

(b) Calculate the net work done on the gas during the cycle.

$$W = -P\Delta V$$

$$W = - (1 \cdot 10^5) (0)$$

$$\text{net work} = 0$$

(c) Was heat added to or removed from the gas during the cycle?

Added Removed Neither added nor removed

Justify your answer.

The heat was neither added or removed because the pressure and volume were conserved. By Plank's rules we can use the equation $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ to show that the heat remained the same.

AP[®] PHYSICS B
2006 SCORING COMMENTARY

Question 5

Overview

This 10-point question was intended to test student understanding of thermodynamics. An unspecified amount of an ideal gas was taken through a cyclic thermodynamic process, which traced out a triangle as shown on the given PV diagram. Students were given the initial temperature of the gas. In part (a) they were to use the ideal gas equation of state along with pressure and volume data taken from the graph to calculate the temperatures in the two other states at the vertices of the cycle. In part (b) students were asked to calculate the net work done on the gas in one cycle. In part (c) they were directed to decide whether the net effect of the cycle was to add heat to the gas or remove heat from the gas, or whether no net heat transfer occurred. For full credit, students had to justify their answer.

Sample: B5A

Score: 10

This student explicitly states in part (a) that PV/T is constant. The entire response is complete and concise.

Sample: B5B

Score: 5

Part (a) earned full credit. In part (b) the work done from state 2 to state 3 is incorrect, so only 2 points were earned there. Part (c) earned nothing since the choice is inconsistent with the sign of the net work obtained.

Sample: B5C

Score: 4

Part (a) earned full credit. In part (b) the student arrives at zero work because the volume does not change after one cycle. Part (c) earned 1 point for a consistent choice, but the justification is incorrect.