CollegeBoard
Advanced Placement Program

Student Performance Q&A:

2006 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2006 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Caren Diefenderfer of Hollins University in Roanoke, Virginia. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

AB Question 1/BC Question 1

What was the intent of this question?

This problem gave two graphs that intersect at $x = 0.15859$ and $x = 3.14619.$ A graphing calculator was required to find these two intersection values. Students needed to use integration to find an area and two volumes. In part (a) students had to find the area of the region bounded by the two graphs. In part (b) students had to calculate the volume of the solid generated by rotating the region about the horizontal line $y = -3,$ a line that lies below the given region. Part (c) tested the students' ability to set up an integral for the volume of a solid generated by rotating the given region around a vertical axis, in this case the y-axis. The given functions could be solved for x in terms of y, leading to the use of horizontal cross sections in the shape of washers and an integral in terms of the variable y. Although no longer included in the *AP Calculus Course Description*, the method of cylindrical shells could also be used to write an integral expression for the volume in terms of the variable x.

How well did students perform on this question?

In part (a) students did quite well. It was somewhat surprising that many students chose to compute this area by partitioning the given region R. In doing so, students made errors in determining the area below the x-axis. Students seemed to be unsure of what to do because the region R included area both above and below the x-axis.

In part (b) the most common error was a rotation about the line $y = 3$ or $y = 0.$ Algebraic errors also occurred in simplifying $(x - 2) - 3.$ In both parts (b) and (c) students often failed to realize that the disk/washer method requires the use of a difference of squares.

One other error that occurred quite frequently involved finding the points of intersection that were needed for the limits of integration. In many cases students lost points because they had presented the limits to fewer than three decimal places. In fewer instances, the limits were never reported on the integral and were never used to determine a final answer.

Many students chose to use the method of cylindrical shells to find the volumes in parts (b) and (c). They were quite successful with this approach in part (c) but had great difficulty with it in part (b).

Students scored better on parts (a) and (c) than on part (b).

The mean score was 4.54 for AB students and 6.06 for BC students (out of a possible 9 points). About 14 percent of AB students and 27 percent of BC students earned all 9 points. Approximately 11 percent of AB students and 3 percent of BC students did not earn any points.

What were common student errors or omissions?

In part (a) most students worked the problem correctly. Common student errors involved presenting the limits to fewer than 3 decimal places, or not stating the limits, either because no intersection points had been found or because the limits were not explicitly stated. Some students incorrectly assumed that the upper limit was π.

In part (b) many students rotated about one of the lines $y = 3$, $y = 0$, or $y = 0.159$. Some students failed to realize that, when R is rotated about $y = -3$, washers are created, and other students did not use a difference of squares to set up this problem. Students continue to have difficulty rotating about an axis other than the x-axis.

When students missed part (c), it was usually because they did not correctly solve the equations $y = \ln(x)$ and $y = x - 2$ for x. Many times students had limits corresponding to the use of washers but failed to convert the equations into functions of y before squaring the terms.

This problem was scored so that the student who made up limits was treated the same as the student who had no limits. Students who used the TRACE function on their calculators to estimate the limits were penalized but not as harshly. In order to determine limits to the desired accuracy, students must use SOLVE or INTERSECT rather than TRACE to find points of intersection.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Continue to require students to practice finding the volume of solids formed by rotating a region about a line other than a coordinate axis.
- Students must know how to find points of intersection using the calculator. The TRACE function does not provide the desired accuracy for intermediate results and should not be used.
- Students must learn to avoid premature rounding.
- Remind students to show their work and use correct mathematical language. Specifically, students must remember to present all information as they set up integrals in the calculator-active problems. For example, neglecting to include the limits of integration can be quite costly.

AB Question 2/BC Question 2

What was the intent of this question?

This problem gave students a function that modeled the rate, in cars per hour, at which cars turn left at a given intersection. In part (a) students had to use the definite integral to find the total number of cars that turned left in a given time period. In part (b) students had to use their graphing calculators to find the time interval during which the rate equaled or exceeded 150 cars per hour, and then compute the average value of the rate over this time interval. Part (c) described a condition that would require the installation of a traffic signal at an intersection, and asked students to decide if a signal was necessary at this particular intersection. Students could do this in several ways. For example, students could recognize that a signal would be required if the number of cars that turn left over a two-hour time interval exceeds 400 cars, and then find an appropriate interval. Or the student might recognize that a signal would be required if the average value of the rate at which cars turn left over any two-hour time interval exceeds 200 cars per hour. Since the rate itself exceeds 200 cars per hour during the interval $13.253 < t < 15.324,$ the average value of the rate will also exceed 200 cars per hour during this time interval of length greater than two hours, and thus a signal would be required.

How well did students perform on this question?

Many students did very well on all three parts of the problem. In particular most students earned both points in part (a). There were a variety of valid approaches to part (c), including some that showed little, if any, explicit calculus notation. Most students used a definite integral to find a two-hour interval with a total number of cars turning left greater than 400. Of these, some constructed a symmetrical interval about the maximum of $L(t)$. Others tested various two-hour intervals until they found an eligible candidate. Others guessed by looking at the graph. Other students searched for two consecutive one-hour intervals with a rate above 200 cars turning left per hour by solving $L(t) \geq 200$.

The mean score was 4.17 for AB students and 5.93 for BC students (out of a possible 9 points). About 10 percent of AB students and 23 percent of BC students earned all 9 points. Approximately 11 percent of AB students and 2 percent of BC students did not earn any points.

What were common student errors or omissions?

In part (b) some students had difficulty with the decimal presentation for the t-interval. Since the problem asked for this interval, correct three decimal presentation was required. Most students were able to write the definite integral for average value and calculate an answer. Some students lost the answer point due to an omission of units. A few students misinterpreted the meaning of the phrase "average value of L over this time interval" and used $L(t) - 150$ as the integrand in the definite integral for average value.

In part (c) most students seemed to have an idea of what they needed to do but did not always communicate well. Many were able to find a two-hour interval on which the total number of cars turning left was greater than 400. Some just estimated and did not show the exact value of the definite integral over their interval. Others calculated a value that was greater than 400 but did not show a comparison that explained their conclusion. A number of students who solved $L(t) > 200$ neglected to state explicitly that they were solving this inequality or they never discussed a two-hour subinterval. A few students confused the arguments by using $L(t) > 200$ and a definite integral.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need to do a better job showing their setups when using the calculator. In particular, when students use calculators to solve an equation or an inequality, they must explicitly write the equation or inequality that they are solving.
- Students need to be more careful with decimals and avoid premature rounding on calculator-active problems. They should be encouraged to store full decimal representations of intermediate calculations in their calculators and then use the stored values in subsequent calculations.
- Teachers need to present students with more nontraditional problems that involve critical thinking such as in part (c) of this problem.

AB Question 3

What was the intent of this question?

This problem required the Fundamental Theorem of Calculus. Students were given the piecewise-linear graph of the function f and were asked about the function g defined as the definite integral of f from 0 to x. It was expected that students would use the graph of f, as well as the area bounded by the graph of f and the x-axis, to answer questions about g, g', and g''. Part (a) asked for the values of $g(4)$, $g'(4)$, and $g''(4)$. Part (b) asked about the behavior of g at $x = 1$. In part (c) the function f is extended in a periodic fashion. Students had to compute $g(10)$, $g(108)$, and $g'(108)$ using the periodic behavior of f.

How well did students perform on this question?

In part (a) students did well. In part (b) students generally earned the answer point but struggled with the correct wording for the justification. In part (c) students correctly gave $g'(108)$ but had difficulty with $g(10)$ and $g(108)$. Students were very comfortable with writing the equation of a tangent line when they had obtained values for $g(108)$ and $g'(108)$.

The mean score was 3.24 (out of a possible 9 points). Only 4 percent of students earned all 9 points, and 22 percent did not earn any points. It is particularly disappointing that students continued to have difficulty with parts (a) and (b) because similar problems have appeared on the Calculus AB exam for at least 10 years.

What were common student errors or omissions?

In part (b) students used vague or unclear language and did not receive the justification point. Many students omitted part (c) altogether. Of those who attempted it, many were unable to determine the value of $g(108)$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need to label their work clearly.
- Students need to learn to justify their results based on the information given in the problem.
- Students need to improve their mathematical communication.
- Students need to improve their ability to interpret graphical data.

AB Question 4/BC Question 4

What was the intent of this question?

This problem presented students with a table of velocity values for rocket A at selected times. In part (a) students needed to recognize the connection between the average acceleration of the rocket over the given time interval and the average rate of change of the velocity over this interval. In part (b) students had to recognize the definite integral as the total change, in feet, in rocket A's position from time $t = 10$ seconds to time $t = 70$ seconds, and then approximate the value of this definite integral using a midpoint Riemann sum and the data in the table. Units of measure were important in both parts (a) and (b). Part (c) introduced a second rocket and gave its acceleration in symbolic form. The students were asked to compare the velocities of the two rockets at time $t = 80$ seconds. The velocity of rocket B could be determined by finding the antiderivative of the acceleration and using the initial condition, or by using the Fundamental Theorem of Calculus and computing a definite integral.

How well did students perform on this question?

The overall performance was good. However, many students seemed much more comfortable with the analytic function given in part (c) than the tabular form used for parts (a) and (b). Students made many arithmetic errors with simple operations on parts (a) and (b), and some students had trouble articulating the meaning of the integral in part (b). Many students forgot to use the initial condition in part (c) or neglected to include a constant of integration with their antiderivative.

The mean score was 3.49 for AB students and 5.29 for BC students (out of a possible 9 points). About 6 percent of AB students and 14 percent of BC students earned all 9 points. Approximately 24 percent of AB students and 7.5 percent of BC students did not earn any points.

What were common student errors or omissions?

In all three parts many students made arithmetic errors. In part (b) some students had trouble articulating the meaning of the given integral. In part (c) many students had trouble finding the antiderivative. Others forgot to include a constant of integration with their antiderivative, and these students could not use the initial condition. Some students who included "$+ C$" then used the initial condition incorrectly. Also in part (c) many students did not explain what value they used from the given table to make their comparison of the speeds of the two rockets.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Give students more practice with functions given in tabular form. Remind them that if they use values from a table in a computation or justification, they should write the values explicitly.
- Give students more practice explaining the meaning of definite integrals.
- Insist that students fully explain how they reach their conclusions.
- Help students gain a better understanding of what an initial condition means and how to use it with both definite and indefinite integrals.
- Help students develop a better understanding of estimation methods for definite integrals. In this instance students had difficulty selecting values for a midpoint Riemann sum.
- To answer the question posed about the speeds of the two rockets, it was necessary to know the speed of rocket *B*. This required students to do some simplification. However, in general, both numeric and algebraic answers do not have to be simplified.
- Remind students to be careful when doing arithmetic and algebra.

AB Question 5

What was the intent of this question?

This problem presented students with a separable differential equation. In part (a) students were asked to sketch its slope field at eight points. Part (b) required solving the separable differential equation to find the particular solution with $f(-1) = 1.$ Students were also asked for the domain of the solution to this differential equation. This is an important consideration when solving any differential equation, and in particular when the differential equation is not defined for all values of the independent and/or dependent variables. Students needed to recognize for this equation that the particular solution must be a differentiable function on an open interval that contains $x = -1$ and does not contain $x = 0.$

How well did students perform on this question?

Students did very well on part (a).

There were many challenges in the first 6 points in part (b), and few students successfully negotiated all of them. Many students earned at least some of the first 5 points, particularly points 4 and 5, but eligibility conditions and algebraic hurdles made the sixth point difficult to earn.

Only a handful of students earned the domain point. Students were not familiar with the key ideas that given a “nice” initial value problem, there is a maximum interval over which there is a guaranteed unique solution to the problem that contains the *x*-value of the initial condition, and that this interval is a natural domain for the solution. Specific information on this topic appears in David Lomen’s article “Solving Separable Differential Equations: Antidifferentiation and Domain Are Both Needed,” which is available on the Calculus AB and Calculus BC Course Home Pages at AP Central®.

The mean score was 3.70 (out of a possible 9 points). Only about 0.1 percent of students earned all 9 points, which demonstrates the difficulty of earning the domain point. Slightly more than 14 percent did not earn any points.

What were common student errors or omissions?

Either forgetting or inappropriately dropping the absolute value signs on the natural logarithm resulted in many student errors. There were quite a few errors with algebraic manipulations of expressions involving the natural logarithm. Also, students failed to notice the contradiction implied by an expression or equation such as $\ln(-1)$ or $-2 = e^c$. Many students did not understand that the solution of the differential equation was valid only for $x < 0$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Emphasize and continue to strengthen routine algebra. This was crucial for separating variables and solving for y.
- Help students understand that when k is negative, $\ln(k)$ has no meaning for real-valued functions. In addition, students must understand that the equation $k = e^c$ has no real solutions when k is nonpositive. When either of these appears in student work, students should recognize that they have made an error.
- Emphasize the reason for the absolute value signs in both $\int \frac{1}{x}\,dx = \ln|x| + C$ and $e^{\ln|x|} = |x|$.
- Emphasize the connection between finding the solution to a differential equation with a given condition and the domain of the solution.

AB Question 6

What was the intent of this question?

This problem gave students the values of $f(0)$, $f'(0)$, and $f''(0)$ for a twice-differentiable function f. In part (a) the function g was defined as the sum of f and an exponential function involving a parameter. Students had to use the chain rule and addition rule for differentiation, and the given information about f, to compute $g'(0)$ and $g''(0)$ in terms of that parameter. Part (b) introduced a function h as the product of f and a cosine function involving the parameter k. Here students had to use the chain rule and product rule to compute the derivative of h, and then use that derivative to write an equation for the line tangent to the graph of h at $x = 0$. Although not asked, it was hoped that the students would make the interesting observation that the equation of the tangent line at $x = 0$ is the same for all values of the parameter k.

How well did students perform on this question?

Students did well in both parts (a) and (b), although it appears that the students were able to handle the trigonometric function in part (b) better than the exponential function in part (a). The students who lost points in part (a) typically had errors in handling the chain rule, a derivative that involves a parameter times x, or a derivative that involves an unspecified function for which only certain information is given. The students who lost points in part (b) typically had errors that involved the chain rule or the product rule. Most students correctly handled the tangent line, but some had difficulty in communicating that the equation for the tangent line was for the function $h(x)$ rather than $f(x)$.

The mean score was 4.79 (out of a possible 9 points). About 15 percent of students earned all 9 points. However, close to 24 percent did not earn any points. This may be due to the fact that this was the last problem on the AB exam, and some students ran out of time and did not have a chance to complete this problem.

What were common student errors or omissions?

The common errors in part (a) included chain rule errors in computing the derivative of e^{ax}, confusing $f'(x)$ with $f'(0)$, and expressing final answers in terms of both the variable x and the constant 0.

The common errors in part (b) included improper use of the chain rule and the product rule. Some students substituted 0 for x prematurely and expressed $h'(x)$ as a function of $f(0)$ and/or $f'(0)$.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers must emphasize the need to communicate carefully. Students must understand that $f(x)$ represents a function and $f(0)$ is a numerical value and not an expression in terms of x.
- Students must practice the chain rule and the product rule with functions that involve parameters.

BC Question 3

What was the intent of this question?

This problem dealt with particle motion in the plane. Students were given the rate of change of the x- and y-coordinates as functions of time and the initial position of a particle at time $t = 2$. Part (a) asked for the acceleration vector and speed of the object at time $t = 2$. Parts (b) and (c) dealt with lines tangent to the curve along which the object moves. In part (b) students had to find the time at which the curve has a vertical tangent line, and in part (c) students had to find a general expression for the slope of the line tangent to the curve at an arbitrary point on the curve. Students were also asked to evaluate the limit of this slope as $t \to \infty$. Part (d) tested the students' ability to use the Fundamental Theorem of Calculus to write an improper integral that represented the value of the horizontal asymptote for the graph of the curve.

How well did students perform on this question?

In part (a) many students lost points because they tried to work with analytic expressions for acceleration and speed rather than computing numerical values with their calculators. In parts (b) and (c) students did fairly well. A majority of students had trouble earning points in part (d).

The mean score was 3.51 (out of a possible 9 points). Slightly more than 1 percent of students earned all 9 points, and more than 17 percent did not earn any points.

What were common student errors or omissions?

The most common mistakes in part (a) involved arithmetic and algebraic errors in finding or simplifying the analytic expressions for acceleration and speed at $t = 2$. These mistakes could have been avoided had students used their calculators to find their answers. Another common error in part (a) was that some students misread the question and found the analytic expression for acceleration but did not evaluate it at $t = 2$.

The most common error in part (b) was setting the slope equal to 0, rather than setting the *x*-component of velocity equal to 0. Many students included $t = -1$ in their solutions in this case.

The most common error in part (c) was that some students could not evaluate the limit. Another less common error was that some students treated $\arcsin(x)$ as the reciprocal of $\sin(x)$.

In part (d) many students used an incorrect expression for the integrand or gave the integrand in an indefinite integral. Many students had trouble finding appropriate limits for the improper integral. A fairly common mistake was using "*c*," the value that they were supposed to find, as one of the limits. Very few students considered an initial condition at all, which is why so few students earned all 9 points on this problem.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Students need practice with deciding when the use of the calculator is appropriate and desirable. Knowing to use the calculator to evaluate a numerical derivative should be second nature.
- Teachers should emphasize the power of the Fundamental Theorem of Calculus, particularly in working with integrals as accumulation functions. Teachers need to help students see the equivalence of the following statements:
$$\int_a^b f(t)\,dt = F(b) - F(a) \text{ and } F(b) = F(a) + \int_a^b f(t)\,dt.$$
- Teachers should emphasize the connections between topics learned in the course. This will help students understand the relationships among accumulation functions, the Fundamental Theorem of Calculus, improper integrals, limits, and horizontal asymptotes. The synthesis of all these ideas was necessary in order for a student to successfully complete part (d).

BC Question 5

What was the intent of this question?

This problem presented students with a differential equation and asked questions about a particular solution satisfying a given initial condition. In part (a) they needed to evaluate the first and second derivatives at the initial condition, using implicit differentiation for the latter computation. In part (b) students could observe that $\frac{dy}{dx} > 0$ when $y = 0$ to help decide if it was possible for the *x*-axis to be tangent to the graph of the particular solution at some point. Part (c) asked for the second-degree Taylor polynomial at $x = -1$, which the students could compute using the initial condition and the results from part (a). In part (d) students needed to use Euler's method with two steps of equal size to approximate the value of the particular solution at $x = 0$. Because the differential equation was not separable, students were not expected to solve the equation in order to answer these questions about the behavior of the particular solution. All questions could be answered by working directly with the differential equation.

How well did students perform on this question?

In part (a) most students successfully substituted x- and y-values to obtain the correct value for $\frac{dy}{dx}$. However, many made arithmetic, algebraic, and differentiation errors when determining $\frac{d^2y}{dx^2}$. Students who attempted part (b) generally did well. Students who attempted part (c) did reasonably well when finding the Taylor polynomial. Most students attempted Euler's method in part (d), but there were many algebraic and arithmetic errors.

This question was split scored with parts (a) and (b) being the AB material (evaluating $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and determining if the x-axis is tangent to the graph of f) and parts (c) and (d) the BC-only material (finding a second-degree Taylor polynomial and using Euler's method). The mean score on parts (a) and (b) was 2.43 (out of a possible 5 points). About 24 percent of students earned all 5 points, and less than 8 percent did not earn any points. The mean score on parts (c) and (d) was 1.90 (out of a possible 4 points). About 18 percent of students earned all 4 points, and about 26 percent did not earn any points. The total mean score was 4.33 (out of a possible 9 points).

What were common student errors or omissions?

In part (a) many students did not use the chain rule as required when finding $\frac{d^2y}{dx^2}$ by implicit differentiation. The most common error in part (b) was failing to recognize that both $y = 0$ and $\frac{dy}{dx} = 0$ are required in order for the x-axis to be tangent to f. In part (c) the most common error was that students presented a Taylor series centered at $x = 1$ or $x = 0$ instead of centered at $x = -1$ as required. Conceptual errors, such as using an incorrect step size or believing that Euler's method involves subtracting the increment in y from the current value instead of adding it, were not nearly as frequent as computational errors in part (d). There was a wide variety of arithmetic and algebra errors, and these occurred most frequently when the student computed the value of the derivative in the second step of Euler's method.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Teachers need to emphasize both the First Derivative Test and the Second Derivative Test so that students understand how to use both tests and how to determine which one is best for a given situation.
- While the tabular approach provided a useful framework for the student to work through the two steps of Euler's method, students who use this approach must also be able to show how they obtain the values they entered into the table. On the other hand, students who use a more conceptual approach, such as the point-slope form of the line, often get bogged down in evaluating the derivative at a particular point and forget to complete the entire process. The latter students would benefit from summarizing their work in tabular form to ensure that all required values are present at each step.

- Students need to develop both specific methods and the conceptual understanding of topics in the AP course. If students are encouraged simply to memorize a formula for a Taylor polynomial or a chart set up for Euler's method without having the conceptual understanding of these approximation methods, the students are not well prepared to explain their work on the AP exam.
- Students need to proofread carefully and learn to check all arithmetic and algebraic computations. Many students lost points for making arithmetic errors when simplifying a correct expression for the second derivative in part (a) and when implementing Euler's method in part (d).

BC Question 6

What was the intent of this question?

This problem dealt with power series. Students were given the power series expansions of two functions, f and g. In part (a) they were asked to find the interval of convergence of the power series for f. Part (b) dealt with the graph of $y = f(x) - g(x)$. Students had to know how to read or compute the values of the first and second derivatives of y at $x = 0$ from the series for f and g. They then needed to use this information to describe the nature of the critical point of y at $x = 0$.

How well did students perform on this question?

In part (a) students did well identifying the radius of convergence and checking the endpoints of their interval. Students had difficulty in explaining the necessary conditions for applying the ratio test, handling the absolute value in the evaluation of their limit, using algebraic manipulations to evaluate their limit, and justifying divergence at the endpoints of their interval.

In part (b) most of the errors in calculating y' and y'' were a result of the inability to handle fractions. Students also attempted to compute $y'(0)$ and $y''(0)$ by evaluating the general term of $f(x) - g(x)$ at $x = 0$. Many students correctly used the Second Derivative Test to justify their conclusion, but several students incorrectly appealed to the First Derivative Test.

The mean score was 3.31 (out of a possible 9 points). While this seems low, it shows significant improvement from the 2005 mean score of 2.47, and it is the first year since 2002 that the mean score on the series question is higher than 3. Only 3 percent of students earned all 9 points, and slightly more than 25 percent did not earn any points.

What were common student errors or omissions?

Common errors in part (a) included misuse of absolute value signs, ignoring the x-term, and forgetting to write an explicit limit statement, as well as numerous algebraic errors. On occasion, absolute value signs appeared and disappeared several times during the evaluation of the limit. The most pervasive error centered on the mishandling of absolute value. Many students determined the correct interval of convergence. However, some students identified the interval as $x > 1$ or $x < 1$, failing to recognize that the interval of convergence in this problem was symmetric about $x = 0$. Some students did not consider the behavior of the series at its endpoints. Others attempted to apply a convergence test to prove divergence at the endpoints.

In part (b) students made arithmetic errors in calculating $y'(0)$. Students also lost points by substituting $x = 0$ in the general term of y' and y'' and believing that was a valid way to compute $y'(0)$ and $y''(0)$.

Some students tried to use the First Derivative Test and did not realize that they had insufficient information to do so. Other students used the Second Derivative Test inappropriately.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

- Mathematical communication is very important. Students must communicate their understanding of the problem and their solution in a clear and correct manner.
- When students wish to appeal to the result of a theorem, they must either state the name of the theorem or give the hypotheses of the theorem and then state the conclusion of the theorem.
- Students must understand that the Alternating Series Test is a test for convergence only; it does not determine whether or not a series diverges.
- Correct algebraic work is very important in working with series problems.
- Students need to understand how to manipulate the absolute value of an expression. Students need more practice and facility with simplifying absolute value expressions in limits. Students need to understand and apply the basic fact that $|-x| = |x|$ and also understand that when $x \neq 0$, $-|x| \neq |x|$.
- Students must understand that infinite series involve an infinite expression and should never truncate an infinite series. Students should be familiar with the ellipsis notation (i.e., $\cdots$) when solving problems that involve infinite series.
- When explaining their conclusions, students should use standard written English and the results of their earlier work. For example, "Since $f'(a) = 0$ and $f''(a) > 0$, the Second Derivative Test tells us that f has a relative minimum at $x = a$" is an example of good work. "Minimum, since $y' = 0$ and $y'' > 0$" is an example of unsatisfactory work.