CollegeBoard
Advanced Placement Program

Commentary on the Instructions for the Free-Response Section of the AP® Calculus Exams

by the AP Calculus Development Committee & Chief Reader

When graphing calculators were first required on the AP Calculus Exams in 1995, the instructions for the free-response section (Section II) of the exams naturally required some significant revisions. With each year's grading experience, new lessons have been learned. The Development Committee and Chief Reader review and revise these instructions annually.

In this report, we include the most recent version of the general instructions for the free-response section, which will be in effect for the 2006 exams. Although the general instructions for the 2006 exams are identical to those for the 2005 exams, we describe some new tips in this document that will help AP students and teachers understand how to interpret these instructions.

It is an excellent idea for AP teachers to go over these free-response instructions with their students well in advance of the exams. By design, the instructions are kept as concise as possible. In a classroom discussion, the teacher has the opportunity to elaborate on and to emphasize key points. The value of this discussion goes beyond the extrinsic motivation of improving students' exam grades. For the most part, the instructions simply stress the importance of communicating written mathematical work clearly. The anonymous graders of the exams play the role of interested readers of the students' work.

Each bulleted instruction is given as it appears on the exams, followed by additional comments that the committee would like to share (*in italics*). Some of these comments are in the form of answers to frequently asked questions (FAQs). Where appropriate, examples from recent official scoring standards are cited to provide additional illustrations.

GENERAL INSTRUCTIONS FOR SECTION II PART A AND PART B

For each part of Section II, you may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

- YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM WITH A PENCIL OR PEN IN THE SPACE PROVIDED FOR THAT PART IN THE PINK EXAM BOOKLET. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.

Comment: *If a student wants the grader to ignore some erroneous work, crossing it out is not only faster, but also a clearer indicator than erasing (some students do not erase as thoroughly as others). Students may use pencil or pen (blue or black ink) for the free-response section.*

FAQ: *What happens if a student actually provides two solutions for the same question and these solutions do not agree?*

Answer: *If it is not clear that the student has abandoned one solution attempt for another, graders are usually instructed to grade both solutions and average the two scores, rounding down to the* *nearest whole number score. For example, if the two solutions are scored 1 and 4 points, respectively, the student is awarded 2 points (the average of 2.5 is rounded down).*

- Show all your work. Clearly label any functions, graphs, tables, or other objects that you use. You will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit.

Comment: *In questions where two or more functions are under discussion (for example, a function and one or more of its derivatives), it is very important for students to make unambiguous references in both their labeling and their prose. In a written explanation, ambiguous references to "the function" or "it" are particularly troublesome when there is more than one "thing" to which these could refer. In general, graders do not infer a specific reference when there is more than one possibility.*

FAQ: *What happens if a student provides a perfectly correct final answer with no supporting work?*

Answer: *In the language of the grading, a "bald" answer is one unadorned by any supporting reasoning or computations. A correct bald answer may earn no credit, or at best, only minimal partial credit. And, of course, an incorrect bald answer has no chance of earning partial credit.*

Comment: *The instruction "Show all your work" is important throughout Section II of the exams. In some problems, students may be specifically reminded to include evidence of the reasoning, strategies, or computations they used to arrive at their answers. The phrasing of this reminder can take several forms, as illustrated below. These reminders add emphasis to the "show your work" instruction. The absence of such a reminder is not an invitation to ignore this instruction. The underlying message is the same for all problems: answers need support to be complete.*

"Justify your answer."

This requires a mathematical argument to back up the claim or conclusion. This is where the application (often with citation) of a theorem, property, or test is generally needed.

Examples: *The existence of a value satisfying a given condition might be justified by showing that the hypotheses are satisfied for the relevant theorem, such as the Intermediate Value Theorem or Mean Value Theorem, or for a test such as the Second Derivative Test. Examples of ways that students can present their work for this type of problem are available in the official scoring standards for 2004 AB3, 2004 AB4/BC4, and 2003 AB4.*

FAQ: *Are sign charts acceptable in justifying either a local or an absolute extremum of a function?*

Answer: *A sign chart is an annotated number line that relates the graphical behavior (increasing/decreasing, concave up/down) of one function with the sign behavior (positive/negative) of another. Sign charts can provide a useful tool to investigate and summarize the behavior of a function. We commend their use as an investigative tool. However, sign charts, by themselves, will not be accepted as a sufficient response when a problem asks for a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more details on this topic, consult "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema," which is available as a .pdf file on the Calculus AB and Calculus BC Home Pages at AP Central®.*

"Give a reason for your answer."
"Explain your reasoning."
"Explain your answer."

*These reminders ask for a basic mathematical connection to back up the answer to a "This or that?" kind of question, especially questions that ask for an interpretation. Another version of "Give a reason for your answer" that might follow a "Yes or no?" question is "Why or why not?" Here are some of the forms of questions that are often followed by such a reminder: Is *blank* increasing or decreasing? concave up or concave down? speeding up or slowing down? OR: Does *blank* happen? How many times does *blank* happen?, etc.*

Examples: *The mathematical (calculus) connection given as a reason might make some reference to the sign or value or behavior of a function or derivative or integral. Examples of ways that students can present their work for this type of problem are available in the official scoring standards for 2004 AB1/BC1, 2003 AB6, and 2003 BC6.*

"Show the analysis (or work) that leads to your conclusion."

This reminder prompts the student to indicate the methods used.

Examples: *In finding an absolute extremum of a function on a closed interval, the student should show that all critical points as well as the endpoints of the interval were considered as candidates. In finding a slope value, the student should show how it was obtained. See the official scoring standard for 2003 AB2 and 2003 AB4 for examples.*

"Show the computations that lead to your answer."

This reminder emphasizes that it is particularly important for the student to show how a final numerical result was obtained.

Examples: *In finding an approximation to a value using a technique such as Euler's method or a Riemann sum or a difference quotient, the student should show the basis of the computation, not just the final numerical result. See the official scoring standard for 2003 AB3 for examples.*

FAQ: *Are there <u>any</u> computations that a student can perform with the calculator without need for showing intermediate computational steps?*

Answer: *Yes. On Part A (the first three free-response problems), students are assumed to have a graphing calculator that can 1) graph a function, 2) numerically solve an equation, 3) numerically compute the value of a derivative at a point, and 4) numerically calculate the value of a definite integral. A student can freely use a calculator for any of these purposes without showing any intermediate work, as long as the student clearly indicates using mathematical language (not calculator syntax) what the calculator is used for (i.e., what we refer to in the directions as the "setup"). With respect to the four capabilities just mentioned, this means: 1) labeling the function, the axes, and the scaling for a graph sketched from the calculator, 2) stating what equation the calculator solver is used for, 3) stating the function and the point at which the numerical derivative was calculated, and 4) stating the definite integral that has been calculated. Note, however, that while a student can sketch a graph from one obtained from a calculator, that graph may not be sufficient as the basis of a mathematical argument.*
